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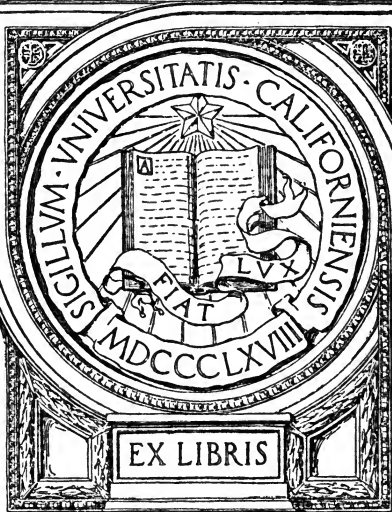
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*J. Arithmetic*

IN MEMORIAM  
FLORIAN CAJORI



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THE *J. Arithmetick*  
**JUVENILE ARITHMETICK**

**AND SCHOLARS' GUIDE,**

**ILLUSTRATED WITH FAMILIAR QUESTIONS,  
AND CONTAINING NUMEROUS EXAMPLES IN**

**FEDERAL MONEY.**

**TO WHICH IS ADDED, A SHORT SYSTEM OF**

**BOOK KEEPING:**

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**BY MARTIN RUTER,**  
President of Augusta College.

**REVISED AND ENLARGED**

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***By Nathan Guilford.***

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**CINCINNATI:**

**PUBLISHED AND SOLD BY N. & G. GUILFORD.**

Stereotyped at the Cincinnati Stereotype Foundry:

**1831.**

**DISTRICT OF OHIO, TO WIT:**

BE IT REMEMBERED, That on the twenty-second day of April, in the year of our Lord one thousand eight hundred and twenty-seven, and in the fifty-first year of the American Independence, MARTIN RUTER, of said District, hath deposited in said office the title of a book, the right whereof he claims as author and proprietor in the words following, to wit:

*"THE JUVENILE ARITHMETICK AND SCHOLAR'S GUIDE: wherein theory and practice are combined and adapted to the capacities of young beginners; containing a due proportion of examples in Federal Money, and the whole being illustrated by numerous questions similar to those of PESTALOZZI, by MARTIN RUTER, A. M.;"*

In conformity to the Act of the Congress of the United States, entitled "An Act for the encouragement of Learning, by securing the copies of Maps, Charts, and Books, to the Authors and Proprietors of such copies during the times therein mentioned; and, also, of the Act entitled "An Act supplementary to an Act entitled an Act for the encouragement of Learning, by securing the copies of Maps, Charts, and Books, to the Authors and Proprietors of such Copies during the times therein mentioned, and extending the benefits thereof to the Arts of designing, engraving, and etching historical and other Prints.

WM. KEY BONL,  
*Clerk of the District of Ohio.*

## RECOMMENDATIONS.

*The following have been selected from the recommendations bestowed upon this work.*

Messrs. Guilfords,—I have examined hastily the "Juvenile Arithmetick," which you sent me, and am of opinion that it possesses some advantages over those generally in use: I particularly refer to the part intended to cultivate in the learner. the habit of going through the solutions mentally. Very respectfully yours,

JOHN E. ANNAN,

Professor of Mathematicks and Natural Philosophy in the Miami University.

Oxford, June 5, 1827.

From a hasty review of Dr. Ruter's Arithmetick, I am inclined to think well of it. The attempt to introduce a *rational* method of instruction in any department of education, is laudible and especially in common schools. This I think the Juvenile Arithmetick is well calculated to do, in that branch of study to which it belongs. The plan of Pestalozzi is excellent, and Dr. Ruter has perhaps imitated it more successfully (by comprizing more in less space) than Mr. Colburn, between whose Arithmetick and this there is however a considerable resemblance. Your's &c.

WM. H. M'GUFFY,

Oxford, June 28, 1827.

Professor of Languages, &c., in the Miami University.

We have examined your 'Juvenile Arithmetick,' and feel a pleasure in recommending it to the schools of our country. We think the general arrangement good, and have no hesitation in saying, that the questions prefixed and appended to the rules, give it superior advantages. Respectfully yours,

JOSEPH S. TOMLINSON,

JOHN P. DURBIN,

March 12, 1828.

Professors in Augusta College.

The Juvenile Arithmetick, from the cursory examination which I have given it, appears to be a *manual of value* for the introduction of youth into the science of numbers. In furnishing a second edition, I wish you success.

ELIJAH SLACK.

I concur most cheerfully in the above opinion.

Cincinnati, April 2, 1828.

S. JOHNSON.

I have used thy compilation of Arithmetick during the last year; and do not hesitate in recommending it to the publick. The questions preceeding the rules, the particular attention to fractions, and the sketch of mensuration give it a decided preference to any other here in use.

JOHN L. TALBERT.

Cincinnati, Fourth mo. 5, 1828.

Having examined the above Arithmetick, I cheerfully concur in the foregoing opinion of its merits.

ARNOLD TRUEDELL.

I have carefully inspected the "Juvenile Arithmetick and Scholar's Guide," by Dr. Ruter, and am of the opinion, it is well calculated and arranged, to conduct the pupil by an easy gradation to a perspicuous conception of the science of numbers. I therefore recommend it to the publick use, particularly in common schools.

September 2, 1827.

SAMUEL BURR,  
Professor of Mathematicks.

A cursory examination of Dr. Ruter's Arithmetick, has convinced me, that the simple and familiar manner in which the learned author unfolds the principles of this science, and adapts them to the understanding of the young learner, can not fail to give his work a *decided preference*, for practical purposes, over those arithmeticks in common use. In my opinion, teachers who adopt it, as well as pupils who study it, will realize satisfactory and highly beneficial results.

S. KIRKHAM,

Author of Grammar in Familiar Lectures.

Pittsburgh, April 2, 1828.

I have examined the system of Arithmetick compiled by Dr. Ruter, and am of opinion that it is well calculated for conveying to youth, a general knowledge of that science in a shorter time, than any I have seen.

G. GARDNER,

March 29, 1828. Teacher of Mathematicks, Mill-Creek Township.

Having examined the *Juvenile Arithmetick*, I have no hesitation in pronouncing it an excellent elementary *School Book*. The rules are judiciously arranged, and peculiarly well adapted to juvenile comprehension: The work contains *multum in parvo*, and I think its publication will be conducive to publick utility.

Hoping its merits will be duly appreciated, I take great pleasure in recommending it to the publick patronage. Yours respectfully,

RICHARD MORECRAFT.

Cincinnati, January 2, 1828.

Teacher.

From my acquaintance with Ruter's Arithmetick, I am convinced that it is well calculated to encourage the student, improve his mind, and prepare him for business.

JOHN LOCKE,

May 16, 1828.

Principal of Cincinnati Female Academy.

Gentlemen—I have examined with some attention the *Juvenile Arithmetick*, &c. by the Rev. Dr. Ruter, and am decidedly of opinion, that it is admirably calculated for conveying to youth with great facility a general knowledge of that important science. The ingenious manner in which the compiler has given an elucidation of *Vulgar Fractions*, together with an exclusion of all extraneous matter, renders it in my estimation a treatise of peculiar merit.

Your obedient servant,

JOHN WINRIGHT,

Cincinnati, September 2, 1827.

Teacher.

## PREFACE.

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THIS ARITHMETICK has been compiled with a view to facilitate the progress of pupils, and lessen the labour of teachers. The questions preceding and following the rules, are designed to lead young learners into habits of *thinking* and calculating; and thus, to prepare them for practical operations. Experience has demonstrated, that, in the instruction of children in any science, it is necessary to excite their entire attention to the subject before them. The latent energies of their minds must be *roused up*, and called forth into action. When this can be *effectually* done, success is rendered certain.—To accomplish this important object, the best method has been found in the frequent use of well selected questions. Though it is a successful course in all juvenile studies, it is particularly so in the science of numbers; and the progress of pupils must be slow without it. The questions in the following pages are thought to be sufficiently numerous for the purposes intended; the rules have been arranged according to the plan of some of the best authors on this subject, and the work is offered to the publick with the hope that it will be useful in the schools of our country.

M. R.

## EXPLANATION OF THE CHARACTERS USED IN ARITHMETICK.

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- $+$  Signifies plus, or addition.
- $-$  Signifies minus, or subtraction.
- $\times$  Denotes multiplication.
- $\div$  Means division.
- $: :: :$  Signifies proportion.
- $=$  Denotes equality.

Thus,  $4+7$  denotes that 7 is to be added to 4.

$5-3$ , Denotes that 3 is to be taken from 5.

$8 \times 2$ , Signifies that 8 is to be multiplied by 2.

$9 \div 3$ , That 9 is to be divided by 3.

$3:2 :: 6:4$ , Shows that 3 is to 2 as 6 is to 4.

$7+9=16$ , Shows that the sum of 7 and 9 is equal to 16.

$\sqrt{\text{or } ^2\sqrt{\text{ }}$  Denotes the Square Root.

$^3\sqrt{\text{ }}$  Denotes the Cube Root.

$^4\sqrt{\text{ }}$  Denotes the Biquadrate Root.

$\text{—}$  This mark, called a Vinculum, shows that the several figures over which it is drawn are to be taken together as a simple quantity.

# ARITHMETICK.



**ARITHMETICK** is the science which treats of the nature and properties of numbers : and its operations are conducted chiefly by five principal rules. These are, *Numeration, Addition, Subtraction, Multiplication, and Division.*

Numbers in Arithmetick are expressed by the following ten digits or characters, namely : 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 cypher.

An Integer signifies a whole number, or certain quantity of units, as one, three, ten. A Fraction is a broken number, or part of a number, as  $\frac{1}{2}$  one half,  $\frac{2}{3}$  two-thirds,  $\frac{1}{4}$  one-fourth,  $\frac{3}{4}$  three-fourths,  $\frac{5}{8}$  five-eighths, &c.

Numeration teaches the different value of figures by their different places, and to express any proposed numbers either by words or characters ; or to read and write any sum or number.



## NUMERATION TABLE.

6	Units.
45	Tens.
332	Hundreds.
76	Thousands.
1	Tens of thousands.
48	Hundreds of thousands.
59	Millions.
44	Tens of millions.
38	Hundreds of millions.
23	Thousands of millions.
65	Ten thousands of millions.
78	Hundred thousands of millions.
9	

Here any figure in the place of units, reckoning from right to left, denotes only its simple value ; but that in the second place denotes ten times its simple value ; and that in the third place, one hundred times its simple value ; and so on, the value of any figure in each successive place, being always ten times its former value. Thus in the number 6543, the 3 in the first place denotes only three ; but 4 in the second place signifies four tens or 40 ; 5 in the third place, five hundred ; and six in the fourth place, six thousand ; which makes the whole number read thus—*six thousand five hundred and forty-three*. The cypher stands for nothing when alone, or when on the left hand side of an integer ; but being joined on the right hand side of other figures, it increases their value in the same ten fold proportion : thus, 50 denotes *five tens* ; and 500 is read *five hundred*.

Though the preceding numeration table contains only twelve places, which render it sufficiently large for young students, yet it may be extended to more places at pleasure.

## EXAMPLE.

Quatrillions.	Trillions.	Billions.	Millions.	Units.
987,654 ;	321,234 ;	567,898 ;	765,432 ;	123,456

Here note, that Billions is substituted for millions of millions : Trillions, for millions of millions of millions : Quatrillions, for millions of millions of millions of millions. From millions, to billions, trillions, quatrillions, and other degrees of numeration, the same intermediate denominations, of *tens, hundreds, thousands, &c.* are used, as from units to millions. And thus, in ascertaining the amount of very high numbers, we proceed from Millions to Billions, Trillions, Quatrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novemdecillions, Vigintillions, &c. all of which answer to millions so often repeated, as their indices respectively require, according to the above proportion.



Write down in figures the following numbers :

Ten.	- - - - -	10
Twenty-one.	- - - - -	21
Thirty-five.	- - - - -	35
Four hundred and sixty-seven.	- - - - -	467
Two thousand three hundred and eighty-nine.	- - - - -	2389
Thirty-four thousand five hundred and seventy	- - - - -	34570
Six hundred and three thousand four hundred.	- - - - -	603400
Seven millions eight hundred and four thousand three hundred and twenty-nine.	- - - - -	7804329
Fifty-eight millions seven hundred and thirty-two thousand one hundred and five.	- - - - -	58732105
Eight hundred and ten millions nine hundred and two thousand five hundred and twelve.	- - - - -	810902512
Three thousand two hundred and three millions six hundred and eight thousand nine hundred and ninety-nine.	- - - - -	3203608999

- Question**
1. What is Arithmetick ?
  2. What are the ten digits by which numbers are expressed ?
  3. What is an integer ?
  4. What is a fraction ?
  5. What are the principal rules by which the operations in Arithmetick are conducted ?
  6. What does Numeration teach ?

## SIMPLE ADDITION.

Simple Addition teaches to put together numbers of the same denomination into one sum ; as 5 dollars, 4 dollars, and 3 dollars, make 12 dollars.

Before the pupil enters upon Addition in the usual way, with figures, it would be useful for him to learn to perform easy operations in his mind. For this purpose let him be exercised in the following questions, or in others which are similar.

1. If you have two cents in one hand and two in the other, how many have you in both ?
2. If you have three cents in one hand, and two in the other, how many have you in both ?
3. If you have five cents in one hand, and two in the other, how many have you in both ?
4. John has six cents, and Robert has three ; how many have they both together ?
5. Charles gave five cents for an orange, and two for an apple ; how many did he give for both ?
6. Dick had four nuts, John had three, and David had two ; how many had they all together ?
7. Henry had five peaches, Joseph had three, and Tom had two, and they put them all into a basket ; how many were there in the basket ?
8. Three boys, Peter John and Oliver, gave some money to a beggar. Peter gave seven cents, John four, and Oliver three. How many did they all give him ?
9. A man bought a sheep for eight dollars, and a calf for seven dollars ; what did he give for both ?
10. A boy gave to one of his companions eight peaches ; to another six ; to another four ; and kept two himself ; how many had he at first ?
11. How many are two and three ?—two and five ?—three and seven ?—four and five ?
12. How many are two and four and one ?
13. How many are three and two and one ?
14. How many are four and three and two ?
15. How many are five and four and three ?
16. How many are four and five and two ?
17. How many are seven and three and one ?
18. How many are eight and four and two ?
19. How many are nine and five and one ?
20. How many are five and six and seven ?
21. How many are four and three and two and one ?
22. How many are two and three and one and four ?
23. How many are five and three and two and one ?\*

\*It is expected that many of these questions will be varied by the teacher, and rendered harder, or easier, or others substituted as the capacity of the pupil may require.

## RULE.

Place the figures to be added, one under another, so that units will stand under units, tens under tens, hundreds under hundreds, &c. Draw a horizontal line under them, and beginning at the bottom of the first column, on the right hand side, that is, at units, add together the figures in that column, proceeding from the bottom to the top. Consider how many tens are contained in their sum, and how many remain besides the even number of tens; place the amount under the column of units, and carry so many as you have tens to the next column. Proceed in the same manner through every column, setting down under the last column its full amount.

## PROOF.

Begin at the top of the sum and add the several rows of figures downwards as they were added upwards, and if the additions in both cases be correct, the sums will agree.

## EXAMPLES.

I.	II.	III.	IV.
1 2	3 2 1	4 0 0 0	5 4 2 2 1 0
2 1	1 2 3	3 1 2 4	1 3 5 4 0 8
3 4	4 1 0	2 3 4 5	3 5 0 2 1 2
1 0	2 0 3	5 2 3 4	2 0 1 3 0 4
<hr/>	<hr/>	<hr/>	<hr/>
7 7	1 0 5 7	1 4 7 0 3	1 2 2 9 1 2 8
<hr/>	<hr/>	<hr/>	<hr/>

V.	VI.	VII.
2 4 0 5 6 7 8	5 0 6 7 8	4 5 0 7 8 9
3 5 4 0 2 1 0	7 6 5 4 3	8 7 6 5 4 3
4 3 2 1 0 2 3	2 0 1 3 4	4 5 0 7 8 9
4 0 6 5 2 4 3	5 6 7 8 7	8 7 6 5 4 3
2 1 2 3 4 5 6	6 5 4 3 2	2 3 4 7 9 8
<hr/>	<hr/>	<hr/>
<hr/>	<hr/>	<hr/>

VIII.	IX.	X.
67898765	45	20000000
4321234	678	3000000
567898	9876	400000
76543	54321	50000
2123	234567	6000
212	987654	700
<hr/> 72866775	<hr/> 9287141	<hr/> 23456700

XL.	XII.	XIII.
24681012	54321231	98765432
42130538	19000310	12345578
71021346	20304986	98765432
20324213	19876540	12345678
98765432	98755432	98765432
12345678	12000987	12345678

## APPLICATION.

1. A boy owed one of his companions 6 cents ; he owed another 8, another 5, and another 9. How much did he owe in all ? Ans. 28 cents.

2. A man received of one of his friends 7 dollars, of another 10, of another 19, and of another 50. How many dollars did he receive ? Ans. 86 dollars.

3. A person bought of one merchant ten barrels of flour, and paid 40 dollars ; of another 20 barrels of cider, for which he paid 60 dollars, and 20 barrels of sugar at 450 dollars ; and of another 95 barrels of salt at 570. How many barrels did he buy, and how much money did he pay for the whole ?

Ans. 145 barrels, and paid 1120 dollars.

4. A had 250 dollars ; B had 375 ; C had 5423 ; D, 64320 ; E, 287432, and F, 4321567. How much would it all make, if put together ? Ans. \$4679367.

**Question.** 1. What does Simple Addition teach ?  
 2. How do you place the numbers to be added ?  
 Where do you begin the Addition ?  
 How do you prove a sum in Addition ?

## SIMPLE SUBTRACTION.

Simple Subtraction teaches to take a less number from a greater of the same denomination, and thus to find the difference between them.

*Questions to prepare the learner for this rule.*

1. If you have seven cents, and give away two; how many will you have left?
2. If you have eight cents, and lose four of them; how many will you have left?
3. A boy having ten cents, gave away four of them; how many had he left?
4. A man owing twelve dollars, paid four of it; how much did he then owe?
5. A man bought a firkin of butter for fifteen dollars, and sold it again for ten dollars; how much did he lose?
6. If a horse is worth ten dollars, and a cow is worth four; how much more is the horse worth than the cow?
7. A boy had eleven apples in a basket, and took out five; how many were left?
8. Susan had fourteen cherries, and ate four of them; how many had she left?
9. Thomas had twenty cents, and paid away five of them for some plums; how many had he left?
10. George is twelve years old, and William is seven; how much older is George than William?
11. Take four from eight; how many will remain?
12. Take three from nine; how many will remain?
13. Take five from ten; how many will remain?
14. Take six from ten; how many will remain?
15. Take six from eleven; how many will remain?
16. Take five from twelve; how many will remain?
17. Take four from thirteen; how many will remain?
18. Take six from fourteen; how many will remain?
19. Take six from fifteen; how many will remain?
20. Take eight from sixteen; how many will remain?
21. Take nine from twelve; how many will remain?
22. Take nine from fourteen; how many will remain?
23. Take three from thirteen; how many will remain?

24. Take eight from seventeen; how many will remain?  
 25. Take nine from sixteen; how many will remain?  
 26. Take nine from eighteen; how many will remain?

## RULE.

Place the larger number uppermost, and the smaller one under it, so that units may stand under units; tens under tens; hundreds under hundreds, &c. Draw a line underneath, and beginning with units, subtract the lower from the upper figure, and set down the remainder.—But when in any place the lower figure is larger than the upper, call the upper one *ten* more than it really is; subtract the lower figure from the upper, considering it as having ten added to it, set down the remainder, and add one to the next left figure of the lower line, and proceed thus through the whole.

## PROOF.

Add the remainder and the less line together, and if the work be right, their sum will be equal to the greater line.

## EXAMPLES.

I.	II.	III.	IV.	V.
23	457	54367	73214	84201
11	215	20154	54876	49983
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
12	242	34213	18338	34218
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>

VI.	VII.
9812030405321	7000000000000
6054123456789	98765432123
<hr/>	<hr/>

VIII.	IX.	X.	XI.
32016	98700	500612	65040032
12045	25290	499521	7000302
<hr/>	<hr/>	<hr/>	<hr/>

XII.	XIII.	XIV.
974865	400000	100000000
863757	7	1
<hr/>	<hr/>	<hr/>

## APPLICATION.

1. A ship's crew consisted of 75 men, 21 of whom died at sea. How many arrived safe in port?

Ans. 54 men.

2. A boy had 100 miles to travel, and went 33 miles the first day. How far had he still to go?

Ans. 67 miles.

3. A tree had 647 apples on it, but 158 of them fell off. How many were there then remaining on the tree?

Ans. 489 apples.

4. A boy put 1000 nuts into a basket and afterwards took out 650. How many were left in the basket?

Ans. 350 nuts.

5. A general had an army of 43250 men, 15342 of them deserted. How many remained?

Ans. 27908 men.

*Question* 1. What does Subtraction teach?

2. How do you place the larger and smaller numbers?

3. What do you do when the lower number is larger than the upper number?

4. How is a sum in subtraction proved?

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*Exercises for the slate under the two preceding Rules.*

1. I saw 15 ladies pass up street, and 8 down street. How many passed both ways?

Ans. 23 ladies.

2. A boy who had 15 buttons upon his jacket, lost off 7 of them. How many were left on?

Ans. 8 buttons.

3. A man bought a barrel of flour for 10 dollars, a barrel of molasses for 29 dollars, and a barrel of rum for 36 dollars. How much did he pay for all the articles?

Ans. 75 dollars.

4. A man bought a chaise for 175 dollars, and to pay for it gave a wagon worth 37 dollars and the rest in money. How much money did he pay.

Ans. 138 dollars.

5. James bought at one time 89 apples, at another 54, at another 60, and at another 75. How many did he in all?

Ans. 278 apples.

6. A merchant bought a piece of cloth containing 489 yards, and sold 365 yards. How many yards had he left?      Ans. 124 yards.

7. Suppose my neighbour should borrow of me at one time 656 dollars, at another 50 dollars, at another 3655 dollars, and at another 5000 dollars; how much should I lend him in all?      Ans. 9361 dollars.

8. Charles has 42 marbles and John has 25. How many has Charles more than John?      Ans. 17 marbles.

9. If Charles give John 200 nuts, and James give him 56, and Joseph give him 195; how many will John have?      Ans. 451 nuts.

10. My friend owed me 150 dollars, but has paid me 90 dollars. How much does he still owe me?      Ans. 60 dollars.

11. If you buy 20 peaches for 40 cents, and sell 15 for 35 cents, how many peaches will you have left, and how much will they cost you?      Ans. 5 peaches, & will cost 5 cents.

12. A person went to collect money, and received of one man 90 dollars; of another 140 dollars; of another 101 dollars, and of another 29 dollars. How much did he collect in all?      Ans. 360 dollars.

13. A man deposited in bank 8752 dollars, and drew out at one time 4234 dollars, at another 1700 dollars, at another 962 dollars, and at another 49 dollars. How much had he remaining in bank?      Ans. 1807 dollars.

14. Gen. Washington was born 1732, and died in 1799. How old was he when he died?      Ans. 67 years.

15. A man owed 11,989 dollars. He paid at one time 2875 dollars; at another 4243; at another 3000 dollars. How much did he still owe?      Ans. 1871 dollars.

16. A man travelled till he found himself 1300 miles from home. On his return, he travelled in one week 235 miles; in the next 275; in the next 325, and in the next 290. How far had he still to go before he would reach home?      Ans. 175 miles.



## SIMPLE MULTIPLICATION.

Simple Multiplication teaches a short method of finding what a number amounts to when repeated a given number of times, and thus performs Addition in a very expeditious manner.

1. What will four apples cost at two cents a piece?
2. What must you give for two oranges, at six cents a piece?
3. What are two barrels of flour worth, at five dollars a barrel?
4. What will three pounds of butter come to, at three cents a pound?
5. If you can walk four miles in one hour, how far can you walk in three hours?
6. If a cent will buy five nuts, how many nuts will four cents buy?
7. What are two barrels of cider worth, at three dollars a barrel?
8. If you give four cents for a yard of tape, how many cents will buy three yards?
9. If I put in your pocket five apples at three different times, how many apples will you have in your pocket? How many are three times five?
10. If four boys have each four apples, how many have they all? How many are four times four?
11. What will six marbles cost at three cents a piece? How many are six times three?
12. A horse has four legs. How many legs have five horses? How many are four times five?
13. I gave six boys four peaches each. How many did I give them all? How many are six times four?
14. How many cents will buy ten marbles if one cost three cents? How many are three times ten?
15. If I can walk three miles in one hour, how far can I walk in six hours?

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Before entering upon this Rule, let the pupil so learn the following table, as to answer with readiness any question implied in it; after which, he will be able to proceed with facility.

## MULTIPLICATION TABLE.

1 twice 1 make 2	3 times 1 make 3	4 times 1 make 4	5 times 1 make 5	6 times 1 make 6	7 times 1 make 7
2	4	2	6	2	8
3	6	3	9	3	12
4	8	4	12	4	16
5	10	5	15	5	20
6	12	6	18	6	24
7	14	7	21	7	28
8	16	8	24	8	32
9	18	9	27	9	36
10	20	10	30	10	40
11	22	11	33	11	44
12	24	12	36	12	48

8 times	9 times	10 times	11 times	12 times
1 make 8	1 make 9	1 make 10	1 make 11	1 make 12
2 16	2 18	2 20	2 22	2 24
3 24	3 27	3 30	3 33	3 36
4 32	4 36	4 40	4 44	4 48
5 40	5 45	5 50	5 55	5 60
6 48	6 54	6 60	6 66	6 72
7 56	7 63	7 70	7 77	7 84
8 64	8 72	8 80	8 88	8 96
9 72	9 81	9 90	9 99	9 108
10 80	10 90	10 100	10 110	10 120
11 88	11 99	11 110	11 121	11 132
12 96	12 108	12 120	12 132	12 144

Though the foregoing table extends no farther than 12, it may be easily continued farther; and if pupils were to extend it, and commit it to memory, as far as 30 or 40, it would afford them great advantage in their progress.

The number to be multiplied is called the multiplicand.

The number which multiplies is called the multiplier.\*

The number produced by the operation is called the product.

\*The multiplier and multiplicand are called Factors.

## CASE I

*When the Multiplier is no more than 12.*

## RULE.

Place the greater number, or multiplicand, uppermost; set the multiplier under it, and beginning with units, multiply all the figures of the multiplicand in succession, carrying one to the next figure for every ten, and setting down the several products, as in Addition. The whole of the last product must be set down.

## PROOF.

Multiply the sum by double the amount of the multiplier, and if the work in both instances be right, the product will be double the amount of the former product.\*

## EXAMPLES.

I.	II.	III.	IV.	V.
234	3201	51000	43201	354610
2	3	4	5	6
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
468	9603	204000	216005	2127660
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
VI.	VII.	VIII.		
453210	3245013	987654321		
7	8	9		
<hr/>	<hr/>	<hr/>		
3172470				
<hr/>	<hr/>	<hr/>		
IX.	X.	XI.		
678987654	321234567	898765432		
9	11	12		
<hr/>	<hr/>	<hr/>		

\*Multiplication may be proved by Division; for if the product be divided by the multiplier, the quotient will be the same as the multiplicand.

## CASE II.

*When the Multiplier is more than 12.*

## RULE.

Multiply each figure in the multiplicand by every figure in the multiplier, and place the first figure of each product exactly under its multiplier; then add the several products together, and their sum will be the answer.

When cyphers occur at the right hand of either of the factors, omit them in multiplying, and annex them to the right hand of the product.\*

When the multiplier is the product of any two whole numbers, the multiplication may be performed by multiplying the sum by one of them, and the product by the other. Thus, if 24 were to be multiplied by 18, (as 6 times 3 make 18,) let it be multiplied by 6, the product by 3, and the answer will be the same as if multiplied by 18.

## EXAMPLES.

$$\begin{array}{r} \text{I.} \\ 43021678 \\ \quad 432 \\ \hline \end{array}$$

$$\begin{array}{r} \text{II.} \\ 87654320 \\ \quad 543 \\ \hline \end{array}$$

$$\begin{array}{r} 86043356 \\ 129065034 \\ 172086712 \\ \hline 18585364896 \end{array}$$

$$\begin{array}{r} 262962960 \\ 350617280 \\ 438271600 \\ \hline 47596295760 \end{array}$$

$$\begin{array}{r} \text{III.} \\ 679100 \\ \quad 32 \\ \hline \end{array}$$

$$\begin{array}{r} \text{IV.} \\ 26043 \\ \quad 34 \\ \hline \end{array}$$

$$\begin{array}{r} \text{V.} \\ 432000 \\ \quad 4300 \\ \hline \end{array}$$

$$\begin{array}{r} 13582 \\ 20373 \\ \hline \end{array}$$

$$\begin{array}{r} 104172 \\ 78129 \\ \hline \end{array}$$

$$\begin{array}{r} 1296 \\ 1728 \\ \hline \end{array}$$

$$21731200$$

$$885462$$

$$1857600000$$

\*Multiplying by 10, add a cypher to the right hand side of the sum, and it is done. Thus, let it be required to multiply 12 by 10, the product will be 120; but if a cypher be added, it will bring the same result. In multiplying by 100, add two cyphers: by 1000, three, &c.

Multiply 18450 by 35.

VI.	VII.	VIII.
18450	18450	18450
7	5	35
<hr/>	<hr/>	<hr/>
129150	92250	92250
5	7	55350
<hr/>	<hr/>	<hr/>
645750	645750	645750

9.	multiply	420	by	7	product	2940
10.		3240		9		29160
11.		54134		18		974412
12.		37990		24		911760
13.		84522		54		4564188
14.		90203		587		52949161
15.		370456		7854		2909561424
16.		7654876		8765		67094988140

## APPLICATION.

1. A man had 29 cows, and his neighbour had five times as many. How many had his neighbour?

Ans. 145.

2. There are twelve barrels of sugar, each containing 256 pounds. How many pounds do they all contain?

Ans. 3072.

3. How far will a man travel in a year, allowing the year to contain 365 days, if he travel 40 miles per day?

Ans. 14600 miles.

4. In one hogshead are 63 gallons;—how many gallons are there in 144 hogsheads?

Ans. 9072.

Q. 1. What does simple Multiplication teach?

2. What is the number to be multiplied, called?

3. What is the number called which is used in multiplying another number?

4. Are the multiplicand and multiplier called by any other names?

5. How do you proceed when the multiplier is no more than 12?

6. When the multiplier is more than 12, how do you proceed?

7. What do you do when cyphers occur at the right hand of either of the factors?
8. How do you proceed when the multiplier is the product of two other numbers?
9. How may sums in Multiplication be proved?

---

## SIMPLE DIVISION.

Simple Division teaches to find how often one number is contained in another, and is a concise way of performing several subtractions.

*Questions to prepare the learner for this rule.*

1. James had 4 apples and John half as many; how many had John?
2. If two oranges cost 6 cents, what does one cost?
3. If you divide 8 apples equally between two boys, how many will each have?
4. What is one half of eight?
5. If you divide 6 nuts equally among 3 boys, how many will each have?
6. What is one third of six?
7. If 12 cherries cost 9 cents, what will 4 cost?
8. A third of 9 is how many?
9. If you divide 16 nuts equally among 4 boys, how many will each have?
10. A fourth of 16 is how many?
11. How many times two are there in six?
12. How many times three in six?
13. How many times four in eight?
14. How many times two in twelve?
15. In nine, how many times three?
16. In eight, how many times two?
17. In ten, how many times five?
18. In twelve, how many times three?
19. In twelve, how many times four?
20. In twenty, how many times five?
21. In eighteen, how many times six?
22. In sixteen, how many times two?

23. In thirty, how many times five?
24. In thirty, how many times six?
25. In twenty-one how many times seven?
26. In twenty-eight, how many times seven?
27. In thirty-six, how many times twelve?
28. In forty-eight, how many times twelve?
29. In forty-eight, how many times sixteen?
30. In fifty-five, how many times eleven?
31. In sixty how many times twenty?
32. In eighty, how many times twenty?
33. In one hundred, how many times twenty?
34. In one hundred and twenty, how many times thirty?
35. In ten, how many times four?

*Answer.* Two times, and two remain.

36. In fourteen, how many times three?

*Answer.* Four times, and two remain.

37. In twenty-five, how many times four?

*Answer.* Six, and one remains.

---

There are in Division four principle parts, viz:

The *dividend*, or number to be divided.

The *divisor*, or number given to divide by.

The *quotient*, or answer, which shows how many times the divisor is contained in the dividend.

The *remainder*, which is any overplus of figure that may remain after the sum is done, and is always less than the divisor.

#### CASE I.

**RULE.**—First, find how many times the divisor is contained in as many figures on the left hand of the dividend as are necessary for the operation, and place the number in the quotient. Multiply the divisor by this number, and set the product under the figures at the left hand of the dividend before mentioned. Subtract this product from that part of the dividend under which it stands, and to the remainder bring down the next figure of the dividend; but if this will not contain the divisor, place a cypher in the quotient, and bring down another figure of the dividend, and so on, untill it will

contain the divisor. Divide this remainder (thus increased) in the same manner as before; and proceed in this manner until all the figures in the dividend are brought down and used.

## PROOF.

Multiply the quotient by the divisor, and to the product add the last remainder, if there be any; if the work is right, the sum will be equal to the dividend.

## EXAMPLES.

I.

Divisor.	Dividend.	Quotient.
3 )	143967182	(47989060
	12	3

23	Proof	143967182
21		

29

27

26

24

27

27

18

18

Remainder. 2

In this example, I find that 3, the divisor, can not be contained in the first figure of the dividend; therefore I take two figures, viz: 14, and inquire how often 3 is contained therein, which I find to be 4 times, and put 4 in the quotient.—

Then multiplying the divisor by it, I set the product under the 14, in the dividend, and find by subtracting that

there is a remainder of two. To this 2, I bring down the next figure in the dividend, viz: 3, which increases the remainder to 23. I then seek how often 3 is contained in 23, and proceed as before. When I bring down the one that is in the dividend, I find that three can not be contained in it, and therefore place a cypher in the quotient and bring down the 8, which makes 18. Finding that 3 is contained 6 times in 18, and that there is no



remainder, I bring down the 2; but as 3 can not be contained in it, I place a cypher in the quotient, and let 2 stand as the last remainder. In proving the sum by Multiplication, the 2 is added. This mode of operation is called **LONG DIVISION**.

II.

2)3456789(1728394

2	2
---	---

14 Proof 3456789

14

5

4

16

16

7

6

18

18

9

8

1

IV.

42)9870(235

84

147

126

210

210

III.

5)6789876(1357975

5	5
---	---

17 Pr 6789876

15

28

25

39

35

48

45

37

35

26

25

1

V.

320(12864016081(40200050

1280

640

640

1608

1600

61

VI.

12 ) 301203 ( 25100

24                      12

61 Pr. 301203

60

12

12

03

VII.

15 ) 218760 ( 14584

15                      15

68                      72920

60                      14584

87                      218760

75

126

120

60

60

VIII.

648 ) 2468098 ( 3808

1944

5240

5184

5698

5184

514

Proof

3808

648

30464

15232

22848

514

2468098

1.	Divide	87654	by 58	Quo.	1511	Rem.	16
2.		456789	679		672		501
3.		3875642	7898		490		5622
4.		98765432	1234		80036		1008
5.		12486240	87654		142		39372
6.		57289761	7569		7569		

*Note.*—When there is one cypher, or more, at the right hand of the divisor, it may be cut off; but when this is done, the same number of figures must be cut off from the right hand of the dividend; and the figures thus cut off, must be placed at the right hand of the remainder.

## EXAMPLES.

$$\begin{array}{r} \text{I.} \\ 6 \overline{)00567434} \mid 10(94572 \end{array}$$

54

27

24

34

30

43

42

14

12

210

$$\begin{array}{r} \text{II.} \\ 18 \overline{)000246864} \mid 593(13714 \end{array}$$

18

66

54

128

126

26

18

84

72

Remainder 12593

*Note.*—In dividing by 10, 100, or 1000, &c. when you cut off as many figures from the dividend as there are cyphers in the divisor, the sum is done; for the figures cut off at the right hand are the remainder, and those at the left are the quotient, as in the following sums:

III.

Quotient.

$$1 \overline{)0}98765(4 \text{ Rem.}$$

Quo.

$$1 \overline{)000}56789(876 \text{ Rem.}$$

IV.

Quotient.

$$1 \overline{)00}123456(78 \text{ Rem.}$$

Quo.

$$1 \overline{)0000}8765(4321 \text{ Rem.}$$

## CASE II.

When the divisor does not exceed 12, seek how often it is contained in the first figure or figures of the dividend, and place the result in the quotient. Then multiply *in your mind* the divisor by the figure placed in the quotient, subtract the product from the figure under which it would properly stand in the former case of division and conceive the remainder, if there be any, to be prefixed to the next figure. See how often the divisor is contained in these, and proceed, as before, till

the whole is divided. This operation is called **SHORT DIVISION**.

**EXAMPLES.**

$$\begin{array}{r} \text{I.} \\ 4 \overline{) 987654321} \end{array}$$

$$\begin{array}{r} \text{II.} \\ 8 \overline{) 123456789} \end{array}$$

Quo. 246913580—1

Quo. 15432098—5

In the first example, I find that 4 is contained twice in 9, and that 1 remains. The 1, I conceive as prefixed to the next figure, which is 8, and they become 18. In 18, I find 4 is contained 4 times, and 2 remain. By prefixing the 2 to the following figure, which is 7, they make 27. In this manner I proceed, setting the result of each calculation in the lower line which is the quotient. In the second example, as 8 can not be contained in 1, take two figures, and proceed as in the first.

$$\begin{array}{r} \text{III.} \\ 9 \overline{) 1023684200} \end{array}$$

$$\begin{array}{r} \text{IV.} \\ 12 \overline{) 1914678987} \end{array}$$

$$\begin{array}{r} \text{V.} \\ 11 \overline{) 6789870062} \end{array}$$

$$\begin{array}{r} \text{VI.} \\ 12 \overline{) 1000001246} \end{array}$$

*Note.*—When the divisor is of such a number that two figures being multiplied together will produce it, divide the dividend by one of those figures, the quotient thence arising, by the other figure, and it will give the quotient required. As it sometimes happens that there is a remainder to each of the quotients, and neither of them the true one, it may be found thus:—Multiply the first divisor by the last remainder, and to the product add the first remainder, which will give the true one.

**EXAMPLES.**

**I.**  
Divide 249738 by 56.

$$8 \overline{) 249738}$$

$$\begin{array}{r} 7 \overline{) 31217} - 2 \end{array}$$

$$4459 - 4$$

$$\begin{array}{r} 8 \\ 4 \\ - \end{array}$$

$$32$$

$$2$$

34 Remainder.

The same done by Long Division.

$$56 \overline{) 249738} [4459$$

$$224$$

$$257$$

$$224$$

$$333$$

$$280$$

$$538$$

$$504$$

34 Remainder.

II.

Divide 1847562324 by 84.

$$12 \overline{) 1847562324} \quad 7 \overline{) 1847562324}$$

$$7 \overline{) 153963527} \quad 12 \overline{) 263937474} - 6$$

$$21994789 - 4$$

$$12$$

48 Rem.

$$21994789 - 6$$

$$7$$

42

$$6$$

Rem. 48

3. Divide 84630986 by 72.

4. " 67860121 by 63.

5. " 12456743 by 96.

6. " 34210390 by 81.

7. " 54697283 by 103.

8. " 75392618 by 113.

*Note.*—In all cases in Division, when there is any remainder, the remainder and divisor form a Vulgar Fraction. Thus, if the divisor be 8 and the remainder 5, they make  $\frac{5}{8}$  or five eighths; or, as in one of the preceding examples, the divisor is 56 and the remainder 34, which make  $\frac{34}{56}$ .

## APPLICATION.

1. A man bought 6 oxen for 318 dollars. How much did he pay a head. Ans. 53 dollars.

2. How much flour at 7 dollars per barrel can be bought for 1512 dollars? Ans. 216 barrels.

3. If 1600 bushels of corn are to be divided equally among 40 men, how much is that a piece?

Ans. 40 bushels.

4. The salary of the President of the United States is 25000 dollars a year. How much is that a day, reckoning 365 days to the year? Ans.  $68\frac{1}{3}$ .

5. A regiment consisting of 500 men are allowed 1000 pounds of pork per day. How much is each man's part? Ans. 2 pounds.

6. If a field of 32 acres produce 1920 bushels of corn, how much is that per acre? Ans. 60 bushels.

7. A prize of 25526 dollars is to be equally divided among 100 men. What will be each man's part?

Ans.  $255\frac{26}{100}$  dollars.

8. How many oxen at 30 dollars a head, may be bought for 38040 dollars? Ans. 1268 oxen.

**Question 1.** What does Simple Division teach?

2. What are the four principal parts of Division?

3. How do you proceed when there is one cypher or more on the right hand of the divisor?

4. How do you proceed in dividing by ten, or a hundred, or a thousand?

5. How do you proceed when the divisor does not exceed 12?

6. When you divide by any number not exceeding 12, what is the operation called?

7. When the divisor is of such a number that two figures multiplied together will produce it?

8. What can be made by placing the remainder of a sum over the divisor? Ans. A Vulgar Fraction.

9. How is a sum in Division proved?

*Promiscuous Exercises on the slate under all the foregoing Rules.*

1. A man bought a cart for 25 dollars, a yoke of oxen for 69 dollars, and a plough for 7 dollars. What did he give for the whole?      Ans. 101 dollars.

2. What will 315 bushels of rye cost at 42 cents a bushel?      Ans. 13230 cents.

3. If my income be 1647 dollars, and I spend 1010 dollars of it, how much do I save?      Ans. 637 dollars.

4. Four boys had gathered 113 bushels of walnuts; in dividing them equally, how many will each have?      Ans. 28 $\frac{1}{4}$  bushels.

5. There are 63 gallons in a hogshead. How many gallons are there in 25 hogsheads?      Ans. 1575 gallons.

6. A merchant bought a stock of goods for 30250 dollars, and sold it again for 40000 dollars. How much did he gain?      Ans. 9750 dollars.

7. A merchant bought at auction, broadcloth for 350 dollars, muslin 97 dollars, linen 1010 dollars, silk 874 dollars, and calico 8 dollars. To what did the whole amount?      Ans. 2339 dollars.

8. There are 328 rows of corn in my field and each row has 169 hills. How many hills are there in the field?      Ans. 55432 hills.

9. John had in his desk 1000 dollars. He took out 120 dollars to pay a debt; he afterwards put in 75 dollars. How much was there in the desk?      Ans. 955 dollars.

10. A farmer has a flock of 60 sheep; one third of them are black and the rest white. How many of them are black?      Ans. 20.

11. A merchant has 50 boxes of raisins with 17 pounds in each box. How many pounds are there in all?      Ans. 850 pounds.

12. There are 4 quarts in a gallon. How many gallons are there in 760 quarts?      Ans. 190 gallons.

13. Ann had a paper of pins which had 600 in it when she bought it, but she used 245 of them. How many are left?      Ans. 355 pins.

14. If I divide 364 cents among 14 boys, how many will each have?      Ans. 26 cents

15. There are 1681 nuts in a basket. James took out 150, Charles 272 and John 1005; after which, Joseph put in 95. How many were there in the basket?

Ans. 349 nuts.

16. A cooper worked 115 days, and made 6 barrels each day. How many barrels did he make?

Ans. 690 barrels.

17. A person has in money 5000 dollars; in bank stock 3500 dollars, and in merchandize 12500 dollars. He intends to divide all this property equally among his 3 sons. What will be the share of each son?

Ans. 7000 dollars.

18. William had 372 pears; he kept 120 of them, and divided the rest between his two sisters. How many did each sister receive?

Ans. 126 pears.

19. There are 10 bags of coffee weighing each 120 pounds, and 12 bags weighing each 135 pounds. What is the weight of the whole?

Ans. 2820 pounds.

20. There are 15 firkins of butter each weighing 49 pounds. The firkins which contain the butter weigh each 7 pounds. How much would the butter weigh without the firkins?

Ans. 630 pounds.

21. A man died, leaving 12426 dollars in cash. He directed in his will that 1000 dollars should be given to his niece; and that the remainder should be equally divided between his two nephews? What is the share of each nephew?

Ans. 5713 dollars.

22. A farmer, who had a farm of 520 acres, bought an adjoining one of 375 acres, and divided the whole equally among his 5 sons. How many acres had each son?

Ans. 179 acres.

23. A merchant bought 5 pieces of linen containing 25 yards each, and 2 pieces containing 24 yards each, and 1 piece containing 26 yards. How many yards were there in the whole?

Ans. 199 yards.

24. Three boys bought 3 baskets, each containing 150 apples, and 2 barrels, each containing 540 apples. They found 219 to be rotten, which they threw away and divided the rest equally among themselves. How many had each for his share?

Ans. 437 apples.



## FEDERAL MONEY.

The denominations of Federal money, or the money of the UNITED STATES, are, Eagle, Dollar, Dime, Cent, and Mill.

**TABLE.**

10 Mills ( <i>m</i> )	make	1 Cent, <i>c</i> .
10 Cents - - -	- - -	1 Dime, <i>d</i> .
10 Dimes - - -	- - -	1 Dollar, <i>D</i> , or \$
10 Dollars - - -	- - -	1 Eagle, <i>E</i> .

In writing Federal Money, it is customary to omit Eagles, Dimes, and Mills, and set down sums in dollars, cents, and parts of a cent. The parts of a cent generally used are, halves, thirds, and quarters. Thus,  $\frac{1}{2}$  is a half;  $\frac{1}{3}$  a third;  $\frac{1}{4}$  a quarter.

### *Exercises for the learner.*

1. How many mills make a cent?—How many half a cent?—How many a cent and a half?—How many two cents?

2. How many halves of a cent make a cent?

3. How many thirds of a cent make a cent?

4. How many fourths of a cent make a half cent?

5. How many fourths make a cent?

6. How many cents make one fourth or quarter of a dollar.

7. How many cents make a half dollar?

8. How many cents make three-fourths of a dollar?

9. How many cents make a dollar?

10. How many dollars and cents in one hundred and ten cents?—How many in two hundred and six cents?—How many in three hundred and forty-eight cents?—How many in five hundred and one cents?

11. If you give a dollar for a book; thirty cents for a slate, and one cent for a pencil, how many cents will you give for the whole?

12. Write down one dollar and eight cents. Two dollars and sixteen cents. Twenty dollars and five cents?

13. Write down three hundred dollars and forty cents.

14. Five hundred eighty-four dollars and fifty cents.

15. Eight hundred sixty dollars and sixty-seven cents.
16. Four thousand eight hundred dollars and two cents.
17. Six hundred thirty-one dollars fifty-six and a fourth cents.
18. Nine hundred and eighty-seven dollars.
19. Thirty-two thousand five hundred dollars eighty seven and a half cents.
20. Ten dollars sixty-eight and three-fourth cents.
21. Twelve dollars ninety-three and three-fourth cents.
22. Twenty dollars thirty-seven and a half cents.
23. Thirty-three dollars thirty-three and a third cents.
24. Sixty dollars sixty-six and two third cents.
25. Read the following sums, viz.

\$8448.87½	\$3450.25	\$47967.91	\$7.10	\$115.33½
\$170.93½	\$19.01	\$85.06½		

## ADDITION OF FEDERAL MONEY.

### RULE.

Begin at the right hand side of the sum, add one row of figures at a time, and carry one for every ten, from the lower denomination to the next higher, as in Simple Addition, until the whole is added. When you come to the last row on the left hand, instead of setting down what remains over ten, twenty, or thirty, &c. set down the full amount.

*Note.*—When there are parts of a cent in a sum, such as halves, &c. find the amount of them in fourths of a cent; consider how many cents these fourths will make, and add them to the first row in the column of cents.—When the parts of a cent are not sufficient to make a cent, place their amount at the right hand of the column of cents, as in the first example; and when the parts of a cent make one cent or more, and some parts remain, but not enough for another cent, the parts thus remaining must be set down in the same way, according to the second example. The proof is the same as in Simple Addition.

## EXAMPLES.

I.	II.	III.	IV.
<i>D. cts.</i>	<i>D. cts.</i>	<i>D. cts.</i>	<i>D. cts.</i>
5432.12½	324.87½	885.90	987654.32
1234.56½	987.43½	125.87½	123456.78
7898.76	720.30	440.40	987654.32
5432.12	842.43½	867.12½	123000.45
3456.78	100.62½	390.97	678987.65
<hr/>	<hr/>	<hr/>	<hr/>
23454.34½	2975.67½	2710.27	2900753.52
<hr/>	<hr/>	<hr/>	<hr/>

## APPLICATION.

1. A man bought a farm in five parcels; for the first, he gave \$250.75; for the second, \$350; for the third, \$475.87½; for the fourth, \$550; and for the fifth, \$600. What was paid for the farm? Ans. 2226.62½

2. A merchant, in buying gave for flour, \$325.43½; for sugar, \$854.25; for molasses, \$520.62½; for coffee, \$944.50; and for cotton, \$6427.12½. What was the sum paid? Ans. \$9071.93½.

3. What is the amount of 10½ cents; 93½ cents; 87½ cents; 50 cts.; 31½ cts.; 43½ cts.; and 11 dollars? Ans. \$14.16½ cents.

4. Gave for an Arithmetick 31½ cents; for a slate, 37½ cents; for quills, 50 cents; for an inkstand, 62½ cents; for a Geography, 1 dollar, and for a History, 87½ cents. How much do they amount to? Ans. \$3.68½ cents.

5. Add \$75212.50, \$644225.75, \$4587220.50, \$90000, and \$5876432.75.

## SUBTRACTION OF FEDERAL MONEY.

## RULE.

Place the smaller sum under the larger, setting the dollars under dollars and cents under cents, and proceed as in Simple Substraction. When there is a fraction, or part of a cent in the upper line of figures, and none in the lower, set it down at the right of the remainder, as

a part of the answer. When there is a fraction in each line, and the upper one is the larger, subtract the lower one from it and set down the difference; but if the lower one is larger than the upper, subtract it from the number that it takes of the fraction to make a cent—add the difference to the upper one, and set down *the amount*. When there is a fraction in the lower line and none in the upper; subtract the fraction from the number that it takes of it to make a cent, and set down the *remainder*. In this case, and likewise when the part or fraction below is larger than the upper one, it is necessary to carry one to the right hand figure of the lower row of cents.

## EXAMPLES.

I.	II.	III.	IV.
<i>D. c.</i>	<i>D. c.</i>	<i>D. c.</i>	<i>D. c.</i>
487.25	587.25	687.31	9000.43
210.10	292.50	599.81	8220.314
<hr/>	<hr/>	<hr/>	<hr/>
\$277.15	\$294.75	\$87.50	\$780.114
<hr/>	<hr/>	<hr/>	<hr/>
V.	VI.	VII.	VIII.
<i>D. c.</i>	<i>D. c.</i>	<i>D. c.</i>	<i>D. c.</i>
695.624	820.434	5978.314	9810600.124
457.874	790.374	4659.934	1987654.684
<hr/>	<hr/>	<hr/>	<hr/>
\$237.75	\$30.064	\$1288.374	\$7822345.444
<hr/>	<hr/>	<hr/>	<hr/>

9. Subtract \$387.20 from \$1000.
10. Subtract \$5871.314, from \$5430.874.
11. Take \$44.874 from 300 dollars.
12. Take \$11000, from \$19876.874.

## APPLICATION.

1. Bought goods amounting to \$1875.624, and having paid \$2350.934; how much remains due?

Ans. 2024.684.

2. My account against my neighbour amounts to \$750.25; and his account against me is \$546.874. How much does he owe me?

Ans. 212.374.

3. Having bought a quantity of goods at \$5425, and sold them at \$3932.68½. How much did I make on the goods? Ans. \$1507.68½.

4. A owes me \$11587.50, but having failed in business, he is able to pay \$9263.62½. How much do I lose? Ans. \$2323.87½.

5. Subtract \$8427.87½, from \$9000. Ans. \$572.12½.

## MULTIPLICATION OF FEDERAL MONEY.

### RULE.

Set the multiplier under the sum, and proceed as in Simple Multiplication, carrying one for every ten from a lower to a higher denomination, until the whole is multiplied. After the sum is done, separate, by a period, the two right hand figures of the product for cents, and the figures at the left hand of the period will be dollars.

*Note.*—When the sum to be multiplied contains a fraction, or part of a cent, multiply it by the multiplier, and consider how many cents are contained in its product. Then multiply the first figure of the cents and add to its product the cents contained in the product of the fraction, and proceed as directed above. In multiplying a fraction, if you find in the product one cent or more, and a remainder not large enough to make another cent, set down the fraction at the right hand of the product, that is under the row of fractions or parts of a cent.—When there is a fraction in the sum, and the multiplier exceeds 12, multiply the sum without the fraction, and afterwards multiply the fraction and add it to the sum.

### EXAMPLES.

I.	II.	III.	IV.	V.
<i>D. c.</i>	<i>D. c.</i>	<i>D. c.</i>	<i>D. c.</i>	<i>D. c.</i>
124.10	830.12½	172.30	2451.62½	275.43½
2	3	4	5	12
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
248.20	2490.37½	689.20	12258.12½	3305.25
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>

VI. D. c	VII. D. c.	VIII. D. c.
164325.11	3120.17	220.314
13	24	37
<hr/>	<hr/>	<hr/>
1314600.88	12480.68	1542.17
1643251.1	62403.4	6609.3
<hr/>	<hr/>	94
2957851.98	74884.08	<hr/>
<hr/>	<hr/>	8151.564

9.	Multiply	\$420.50	by	2	Ans.	\$841.00
10.	"	519.75	by	3	Ans.	1559.25
11.	"	99.624	by	4	Ans.	398.50
12.	"	75.314	by	5	Ans.	376.564
13.	"	62.124	by	6	Ans.	372.75
14.	"	750.25	by	7	Ans.	5251.75
15.	"	330.124	by	8	Ans.	2641.00
16.	"	248.874	by	9	Ans.	2239.874
17.	"	95.934	by	12	Ans.	1151.25
18.	"	24.17	by	10	Ans.	241.70*
19.	"	37.50	by	28	Ans.	1050.00
20.	"	58.934	by	36	Ans.	2121.75
21.	"	9876.624	by	208	Ans.	2054338.00

## APPLICATION.

- How much will 18 barrels of flour cost, at 3 dollars per barrel? Ans. 54 dollars.
- What will 35 pounds of coffee cost, at 20 cents per pound? Ans. 7 dollars.
- Sold 87 barrels of flour, at \$3.124 per barrel. What was the amount? Ans. \$271.874.
- Bought 160 acres of land, at \$1.25 per acre.—What did the whole cost? Ans. 200 dollars.
- What will 225 bushels of apples cost, at 624 cents per bushel? Ans. 140.624.
- What will 580 bushels of salt cost, at \$1.124 per bushel? Ans. \$652.50.

\*In multiplying by 10, when there is no fraction in the sum, it is necessary to add a cypher to the right hand of the sum, placing the period that separates cents from dollars one figure farther towards the right hand, and the sum is done. In multiplying by 100, add two cyphers: by 1000, three, &c.

## DIVISION OF FEDERAL MONEY.

## RULE.

Proceed as in Simple Division. When the sum consists of dollars and cents, the two right hand figures of the quotient will be cents. When there is a remainder, multiply it by 4, adding the number of fourths that are in the fraction of the sum (if there be any) to its product; then divide this product by the divisor, and its quotient will be fourths, which must be annexed to the quotient of the sum. When the sum consists of dollars only, if there be a remainder, add two cyphers to it; then divide by the divisor as before, and its quotient will be cents, which must be added to the quotient of the sum. When the sum is in dollars, and the divisor is larger than the dividend, add two cyphers to the dividend—then divide, and the quotient will be in cents.

## EXAMPLES.

I.	II.	III.	IV.
<i>D. c.</i>	<i>D. c.</i>	<i>D. c.</i>	<i>D. c. D. c.</i>
2)120.50	4)8000.00	5)580.75	7)34.49(12.07
<hr/> 210.25	<hr/> 2000.00	<hr/> 116.15	7
			<hr/> 14
			14
			<hr/>
			49
			49
			<hr/>
V.	VI.	VII.	
<i>D. c.</i>	<i>D. c. D. c.</i>	<i>D. c. D. c.</i>	
9)27.81(3.09	12)144.60(12.05	36)162.36(4.51	
27	12	144	
<hr/>	<hr/>	<hr/>	
81	24	183	
81	24	180	
<hr/>	<hr/>	<hr/>	
	60	36	
	60	36	
	<hr/>	<hr/>	

## VIII.

*D. c. D. c.*  
36)1234.72(34.294

108

154

144

107

72

352

324

28

4

36)112(3

108

4

## IX.

*D. c. D. c.*  
44)87654.32(1992.144

44

436

396

405

396

94

88

63

44

192

176

16

4

44)64(1

44

20+

10.	Divide	\$640
11.	"	717.124
12.	"	246.25
13.	"	687.20
14.	"	980
15.	"	87654
16.	"	1284
17.	"	40000
18.	"	976.874
19.	"	1334.374
20.	"	9876.44
21.	"	89786.54

by	12
by	8
by	9
by	12
by	34
by	128
by	112
by	188
by	225
by	212
by	345
by	374



## APPLICATION.

1. Divide 400 dollars, equally, among 20 persons.  
What will be the portion of each person? Ans. \$20.

2. Divide 1728 dollars, equally among 12 persons.  
What does each one of them share? Ans. \$144.

3. If 240 bushels cost 420 dollars; what is the cost of  
one bushel at the same rate? Ans. \$1.75.

*Promiscuous Examples.*

1. What will the following sums amount to, when added together, namely:—

\$124.62½; \$248.87½; \$342.40; \$9850.25.  
and \$20.31½? Ans. \$10586.46½.

2. If my estate is worth 12870 dollars, and I meet  
with losses amounting to \$4364.50, how much shall I  
have left? Ans. \$8505.50.

3. A merchant enters into a trade by which he re-  
ceives \$1324.62½ per year, for four years; how much  
is his whole gain? Ans. \$5298.50.

4. An estate of 98740 dollars is to be divided, equally,  
between 8 heirs; what will each receive?

Ans. \$12342.50.

A bought of B,

1 barrel of sugar at	-	-	\$24.50
1 chest of tea,	-	-	60.00
1 hogshead of salt,	-	-	3.75
20 yards of cloth,	-	-	15.00
1 barrel of flour,	-	-	3.87½

Ans. \$107.12½.

Q. 1. What are the denominations of Federal Money?

2. How many mills make a cent?

3. How many cents make a dime?

4. How many dimes make a dollar?

5. How many dollars make an eagle?

6. How are the denominations generally used in  
writing Federal Money, and in reckoning?

7. Where is Federal Money used as a currency?

Answer. In the United States of North America.

*Exercises for the slate in Federal Money.*

1. What is the sum of 50 cents and 5 dimes?  
Ans. 1 dollar.
2. A man owed \$36.46.5, and paid \$27.69.6; how much did he then owe?  
Ans. \$8.76.9.
3. A man laboured 6 months at 25 dollars 6 cents and 5 mills per month. How much did his wages amount to?  
Ans. \$150.39.
4. If you divide \$35001.50 equally among 125 men, how many dollars will each have?  
Ans. \$280.01.2.
5. Bought an umbrella for four dollars, a penknife for 37 cents and 5 mills, a hat for 3 dollars and 64 cents, a cane for one dollar and 5 dimes, and a book for one dollar and a quarter. How much did he pay for all these articles.  
Ans. \$10.19.
6. What will 1473 peaches come to at 5 mills a piece?  
Ans. 7 dollars 36 cents 5 mills, or \$7.36.5.
7. A lady bought 8 yards of silk for 10.37.6. How much is that a yard?  
Ans. \$1.29.7.
8. A boy sold a top for a dollar, and gave away 124 cents of the money. How much had he left?  
Ans. 874 cents.
9. Add two mills, two cents two dimes, and two dollars together.  
Ans. \$2.22.2.
19. You borrow \$536.15 and pay \$236.18; how much remains unpaid.  
Ans. 299.97.
11. Going a journey I took two 50 dollar bills to bear my expenses, which were as follows, viz: Stage fare eighteen dollars; board, nine dollars and fifty cents; carrying trunk, seventy-five cents; private conveyance at one time, six dollars and 374 cents; and at another, seven dollars. How much had I left, on my return home?  
Ans. 58.374.
12. From five dollars take one mill?  
Ans. \$4.99.9.
13. From 4 dollars take 3 dollars 99 cents and 9 mills.  
Ans. \$0.00.1.
14. What will 2640 bushels of oats cost at 124 cents per bushel?  
Ans. \$330.
15. A man paid \$27.60 for 12 yards of cloth. How much was the cloth a yard?  
Ans. \$2.30.

16. If John spends 6 cents a day, how many would he spend in a year, or 365 days? Ans. \$21.90.

17. I bought a wagon for 70 dollars, and paid \$9.12½ for repairing it, and sold it again for \$75.62½. Did I make or lose by the bargain? and how much?

Ans. Lost \$3.50.

18. I bought at a store a pair of gloves for 75 cents, a vest pattern for \$1.12½, two yards of cloth for a coat at \$2.87½ per yard, and trimmings for 18½ cents; and to pay the bill, I gave the merchant an eagle. How much should he give me in change? Ans. \$2.18½.

19. Three men sold their marketing for 10 dollars. What would be the share of each, if the money were divided equally among them? \$3.33½

20. A lady went a shopping with the sum of 55 dollars and her purchases amounted to 29 dollars and 62½ cents; how much had she remaining? Ans. \$25.37½.

21. If a person pay for board \$3.50 a week, how much will he pay in 26 weeks? Ans. \$91.00.

22. A person bought a box of tea weighing 14 pounds for \$17.50; how much is it a pound? Ans. \$1.25.

23. A gentleman rents a house for 650 dollars a year; how much is it a month? Ans. \$54.16.6.

24. A person has due to him the following sums of money, viz: 574 dollars 6½ cents; 425 dollars 25 cents; 175 dollars 68½ cents; 341 dollars 87½ cents; 1021 dollars 12½ cents. How much is due to him in all?

Ans. \$2538.00.

25. Bought 35 yards of cloth at 2 dollars and 6½ cents a yard; 51 pair of shoes at 87½ cents a yard; 10 dozen of buttons, at 18½ cents a dozen, and 75 hats, at \$3.25 a piece. To how much did the whole amount?

Ans. 362.43½.

26. Four men about to descend the Ohio river in a boat, laid in the following provisions, viz: 26 loaves of bread, at 6½ cents a loaf: 41 pounds of ham, at 12½ cents a pound; 17 pounds of coffee at 18½ cents a pound, 15 pounds of cheese at 9 cents a pound, and paid for sundry other articles \$6.43½. How much did the whole amount to, and what was the share of each?

Ans. \$17.72. Each man's share \$4.43.

# TABLE

## OF

### MONEY, WEIGHTS, MEASURES, &c.

#### ENGLISH MONEY.

A table of Federal Money has already been given.

The denominations of English Money are pound, shilling, penny, and farthing.

4 farthings ( <i>qr.</i> )	make	1 penny	<i>d.</i>
12 pence - - -	- - -	1 shilling	<i>s.</i>
20 shillings - - -	- - -	1 pound	<i>£.</i>

✂ Farthings are written as fractions, thus:

- $\frac{1}{4}$  one farthing.
- $\frac{2}{4}$  two farthings, or a half-penny.
- $\frac{3}{4}$  three farthings.

#### PENCE TABLE.

<i>d.</i>		<i>s.</i>	<i>d.</i>
20	pence make	1	8
30	" "	2	6
40	" "	3	4
50	" "	4	2
60	" "	5	0
70	" "	5	10
80	" "	6	8
90	" "	7	6
100	" "	8	4
110	" "	9	2
120	" "	10	0
240	" "	20	0

#### SHILLING TABLE.

<i>s.</i>		<i>£.</i>	<i>s.</i>
20	shillings make	1	0
30	- -	1	10
40	- -	2	0
50	- -	2	10
60	- -	3	0
70	- -	3	10
80	- -	4	0
90	- -	4	10
100	- -	5	0
110	- -	5	10
120	- -	6	0
130	- -	6	10

## TROY WEIGHT.

By this weight, jewels, gold, silver, and liquors are weighed.

The denominations of Troy Weight are pound, ounce, pennyweight, and grain.

24 grains ( <i>gr.</i> )	make	1 pennyweight	<i>dwt.</i>
20 pennyweights	- -	1 ounce	<i>oz.</i>
12 ounces	- - -	1 pound	<i>lb.</i>

## AVOIRDUPOIS WEIGHT.

By this weight are weighed things of a coarse drossy nature, that are bought and sold by weight; and all metals but silver and gold.

The denominations of Avoirdupois Weight are ton, hundred weight, quarter, pound, ounce, and dram.

16 drams, ( <i>dr.</i> )	make	1 ounce	- -	<i>oz.</i>
16 ounces	- -	1 pound	- -	<i>lb.</i>
28 pounds	- -	1 quarter of a cwt.		<i>qr.</i>
4 quarters, or 112 lb.		1 hundred weight		<i>cwt.</i>
20 hundred weight.		1 ton.	- -	<i>T.</i>

## APOTHECARIES WEIGHT.

By this weight apothecaries mix their medicines, but buy and sell by Avoirdupois Weight.

The denominations of Apothecaries Weight are pound, ounce, dram, scruple, and grain.

20 grains ( <i>gr.</i> )	make	1 scruple	<i>ʒ</i>
3 scruples	- - -	1 dram	<i>ʒ</i>
8 drams	- - -	1 ounce	<i>ʒ</i>
12 ounces	- - -	1 pound	<i>lb</i>

## LONG MEASURE.

Long measure is used for lengths and distances.

The denominations of Long Measure are degree, league, mile, furlong, pole, yard, foot, and inch.

12 inches ( <i>in.</i> )	make	1 foot	-	-	<i>ft.</i>
3 feet	-	1 yard	-	-	<i>yd.</i>
5½ yards, or 16½ feet		1 rod, pole, or perch	-	-	<i>P.</i>
40 poles (or 220 yds.)		1 furlong	-	-	<i>fur.</i>
8 furlongs (or 1760 yds.)		1 mile	-	-	<i>M.</i>
3 miles	-	1 league	-	-	<i>L.</i>
60 geographick, or 69½ statute	}	miles	1 degree	-	<i>deg.</i>

*Note.*—A hand is a measure of 4 inches, and used in measuring the height of horses.

A fathom is 6 feet, and used chiefly in measuring the depth of water.

### CUBICK, OR SOLID MEASURE.

By Cubick, or Solid Measure, are measured all things that have length, breadth and thickness.

Its denominations are, inches, feet, ton, or load, and cord.

1728 inches	-	make	-	1 cubick foot.
27 feet	-	-	-	1 yard.
40 feet of round timber or 50 feet of hewn timber.	}	-	-	1 ton or load.
128 solid feet, i. e. 8 in length, 4 in breadth, and 4 in height.		-	-	1 cord of wood.

### LAND, OR SQUARE MEASURE.

This measure shows the quantity of lands.

The denominations of land Measure are acre, rood, square perch, square yard, and square foot.

144 square inches	make	1 square foot	<i>ft.</i>
9 square feet	-	1 square yard	<i>yd.</i>
30½ square yards	-	1 square perch	<i>P.</i>
40 square perches	-	1 rood	<i>R.</i>
4 roods	-	1 acre	<i>A.</i>
640 acres	-	1 mile	<i>m.</i>

## CLOTH MEASURE.

By this measure cloth, tapes, &c. are measured.

The denominations of Cloth Measure are English ell, Flemish ell, yard, quarter of a yard, and nail.

4 nails ( <i>na.</i> )	make	1 quarter of a yard	<i>qr.</i>
4 quarters	- -	1 yard	<i>yd.</i>
3 quarters	- -	1 ell Flemish	<i>E. Fl.</i>
5 quarters	- -	1 ell English	<i>E. E.</i>
6 quarters	- -	1 ell French	<i>E. F.</i>

## DRY MEASURE.

This measure is used for grain, fruit, salt, &c.

The denominations of Dry Measure are bushel, peck, quart, and pint.

2 pints ( <i>pt.</i> )	make	1 quart	- -	<i>qt.</i>
8 quarts	- - -	1 peck	- -	<i>pe.</i>
4 pecks	- - -	1 bushel	-	<i>bu.</i>

## WINE MEASURE.

By Wine Measure are measured Rum, Brandy, Perry, Cider, Mead, Vinegar and Oil.

Its denominations are pint, quart, gallon, hogshead, pipe, &c.

2 pints ( <i>pt.</i> )	make	1 quart	-	<i>qt.</i>
4 quarts	- - -	1 gallon	-	<i>gal.</i>
42 gallons	- - -	1 tierce	-	<i>tier.</i>
63 gallons	- - -	1 hogshead	-	<i>hhd.</i>
2 hogsheads	- - -	1 pipe or butt	-	<i>P. or B.</i>
2 pipes	- - -	1 ton	-	<i>T.</i>

## ALE, OR BEER MEASURE.

The denominations of this measure are pint, quart, gallon, barrel, &c.

2 pints ( <i>pt.</i> )	make	1 quart - -	<i>qt.</i>
4 quarts - -	- - -	1 gallon - -	<i>gal.</i>
8 gallons - -	- - -	1 firkin of ale -	<i>fir.</i>
2 firkins - -	- - -	1 kilderkin - -	<i>kil.</i>
2 kilderkins - -	- - -	1 barrel - -	<i>bar.</i>
1½ barrels, or 54 gallons		1 hogshead of beer	<i>hhd.</i>
2 barrels - -	- - -	1 puncheon - -	<i>pun.</i>
3 barrels, or 2 hogsheads		1 butt - -	<i>butt.</i>

## TIME.

The denominations of Time are year, month, week, day, hour, minute and second.

60 seconds ( <i>sec.</i> )	make	1 minute -	<i>min.</i>
60 minutes - -	- - -	1 hour -	<i>H.</i>
24 hours - -	- - -	1 day -	<i>D.</i>
7 days - -	- - -	1 week -	<i>W.</i>
52 weeks, 1 day, and 6 hours,	}	1 year -	<i>Y.</i>
or 365 days, and 6 hours,			
12 months ( <i>mo.</i> ) - -	- - -	1 year -	<i>Y.</i>

*Note.*—The six hours in each year are not reckoned till they amount to one day; hence, a common year consists of 365 days, and every fourth year, called leap year, of 366 days.

The following is a statement of the number of days in each of the twelve months, as they stand in the calendar or almanack:

The fourth, eleventh, ninth, and sixth,  
Have thirty days to each affix'd:  
And every other thirty-one,  
Except the second month alone,  
Which has but twenty-eight in fine,  
Till leap year gives it twenty-nine.

## MOTION.

60 seconds "	make	1 prime minute,	'
60 minutes - -	- - -	1 degree - -	°
30 degrees - -	- - -	1 sign - -	♊.
12 signs, or 360 degrees	} The whole great circle of the Zodiack.		



# REDUCTION.

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Reduction teaches to change numbers of one denomination into those of other denominations, retaining the same value. Its operations are performed by Multiplication and Division. When performed by Multiplication, it is called Reduction Descending, when performed by Division, it is called Reduction Ascending.

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## QUESTIONS.

1. How many farthings will it take to make two pence? How many pence to make two shillings?—How many shillings to make two pounds?

2. How many gills to make three pints?—How many pints to make three quarts?—How many quarts to make three gallons?

3. How many quarts to make four pecks?—How many pecks to make four bushels?

4. How many pence are there in eight farthings?—How many shillings in twenty-four pence?—How many pounds in forty shillings?

5. How many pints in twelve gills?—How many quarts in six pints?—How many gallons in twelve quarts?

6. How many pecks in thirty-two quarts?—How many bushels in sixteen pecks?

7. How many pounds and shillings in thirty shillings? How many shillings and pence in thirty pence?

---

## REDUCTION DESCENDING.

### RULE.

Multiply the numbers in the highest denomination given, by the number that it takes of the next less denomination to make one of that greater; and thus proceed until you shall have multiplied each higher denomination by the number that it takes to form the next lower, until you come to the lowest of all.

## PROOF.

Descending and Ascending Reduction prove each other.

## SIMPLE EXAMPLES.

I.

Reduce 25 pounds to shillings

25

20 shillings in a pound.

---

500 shillings.

Ans. 500 shillings.

II.

Reduce 50 shillings to pence.

50

12 pence in a shilling,

---

600 pence.

Ans. 600 pence.

III.

Reduce 15 pence to farthings.

15

4 farthings in a penny.

---

60 farthings.

Ans. 60 farthings.

IV.

Reduce 10 tons to hundred weights.

10

20 hundreds in a ton.

---

200 hundreds.

Ans. 200 cwt.

v.

Reduce 36 pounds to ounces.

36

16 ounces in a pound.

---

216

36

---

576 ounces.

Ans. 576 ounces.

6. Reduce 70 miles to furlongs.      Ans. 560 fur.  
 7. Bring 30 furlongs to rods.      Ans. 1200 rods.  
 8. Bring 20 rods to feet.      Ans. 330 feet.  
 9. Bring 24 feet to inches.      Ans. 288 inches  
 10. Reduce 32 acres to roods.      Ans. 128 roods.  
 11. Bring 24 square perches to square yards.  
     Ans. 726 square yards.  
 12. Reduce 10 hogsheads to gallons.      Ans. 630 gal.  
 13. Bring 25 gallons to pints.      Ans. 200 pints.  
 14. Reduce 23 bushels to pecks.      Ans. 92 pecks.  
 15. Bring 12 pecks to pints.      Ans. 192 pints.  
 16. Reduce 15 years to months.      Ans. 180 months.  
 17. Bring 75 days to hours.      Ans. 1800 hours.  
 18. Bring 24 hours to minutes.      Ans. 1440 minutes.  
 19. Bring 10 signs to degrees.      Ans. 600 degrees.

## COMPOUND EXAMPLES.

I.

	£.	s.	d.	qrs.	
In	15	17	11	3	how many farthings?
	20 shillings in a pound.				

---

317 shillings.

12 pence in a shilling.

---

3815 pence.

4 farthings in a penny.

---

15263 farthings.

---

*Note.*—In multiplying by 20, I added in the 17 shillings, by 12, the 11 pence; and by 4, the 3 farthings; and this must be observed in all similar cases.

To prove this sum, let the order of it be changed, and it will stand thus: in 15263 farthings how many pounds?

$$4)15263$$

$$12)3815+3 \text{ quarters.}$$

$$2)0)31|7+11 \text{ pence.}$$

$$\text{£}15 \text{ 17s. 11 d. 3 qrs. Ans.}$$

In reducing Federal Money from a higher to a lower denomination, it is only necessary to annex as many cyphers as there are places from the denomination given to that required; or if the given sum be of different denominations, annex the figures of the several denominations in their order, and continue with cyphers, when the sum requires it, to the denomination intended.

2. Thus, in 7 eagles, 3 dollars, how many mills?

Ans. 73000.

3. In 85 dollars, how many mills?

Ans. 85000.

4. In 574 eagles, how many dollars?

Ans. 5740.

5. In 469 dollars, how many cents?

Ans. 46900.

6. In 844 dollars, 75 cents, how many mills?

Ans. 844750.

7. In 1000 dollars, how many mills?

Ans. 1000000.

8. In 25 dollars, 47 cents, 8 mills; how many mills?

9. In 29 guineas at 28s. each, how many pence?

Ans. 9744.

10. In 20 acres, 29 poles, or perches, how many square perches?

Ans. 3229.

11. How many solid feet in 30 cords of wood?

Ans. 3840.

12. How many grains in 100 lbs.—Troy Weight?

Ans. 576000.

13. How many lbs. in a ton:—Avoirdupois Weight?

Ans. 2240.

14. In 27 lbs.—Apothecaries Weight; now many grains?      Ans. 155520.  
 15. In 30 yards, how many nails?      Ans. 480.  
 16. In 360 degrees, being the distance round the world, how many inches, allowing  $69\frac{1}{2}$  miles to a degree?      Ans. 1,587,267,200.  
 17. How many pints are there in one tun of wine?      Ans. 2016.  
 18. How many half pints in one hogshead of beer?      Ans. 864.  
 19. How many pints in 400 bushels?      Ans. 25600.  
 20. How many seconds in 80 years of 365 days each?      Ans. 2,522,880,000.  
 21. How many yards in 4567 miles?      Ans. 8037920.  
 22. In £20 17s., how many pence and half pence?      Ans. 5004 pence, and 10,008 half pence.

### REDUCTION ASCENDING.

#### RULE.

Divide the figure or figures in the lowest denomination, by so many of that name as make one of the next higher; and continue the division until you have brought it into that denomination which your question requires.

In reducing Federal Money from a lower to a higher denomination, nothing more is necessary than to cut off so many places on the right hand side of the sum, as there are denominations lower than the one required. Thus, 98765 mills are reduced to dollars, cents, and mills, by cutting off one figure for mills, two more for cents, and the remaining figures being dollars, the amount is \$98|76|5—or ninety-eight dollars, seventy-six cents, five mills.

#### SIMPLE EXAMPLES.

1. How many dollars are there in 8000 mills?      8|00|0      Ans. 8.  
 2. In 487525 cents, how many dollars and cents?      4875|25      Ans. \$4875.25.

3. In 999888 mills, how many dollars, cents and mills?  
 $999\overline{)888}$  Ans. \$999.88.8.

4. In 19200 farthings, how many pounds?

$$\begin{array}{r} 4\overline{)19200} \\ \hline \end{array}$$

$$\begin{array}{r} 12\overline{)4800} \\ \hline \end{array}$$

$$\begin{array}{r} 20\overline{)400} \\ \hline \end{array}$$

Ans. 20 pounds.

5. In 480 nails, how many yards?

$$\begin{array}{r} 4\overline{)480} \\ \hline \end{array}$$

$$\begin{array}{r} 4\overline{)120} \\ \hline \end{array}$$

30 Ans.

#### COMPOUND EXAMPLES.

6. In 52300 farthings, how many pounds?

$$\begin{array}{r} 4\overline{)52300} \\ \hline \end{array}$$

$$\begin{array}{r} 12\overline{)13075} \\ \hline \end{array}$$

$$\begin{array}{r} 2\overline{)1089+7} \\ \hline \end{array}$$

Ans. £54 9s. 7d.

7. In 8828 lbs. Avoirdupois Weight, how many tons?

Ans. 3 tons. 18cwt. 3qrs. 8lbs.

8. In 524 lbs. Avoirdupois Weight, how many cwt.

&c.

Ans. 4 cwt. 2 qrs. 20lbs.

9. In 253440 grains, Troy Weight, how many lbs?

Ans. 44.

10. In 155520 grains, Apothecaries Weight, how many pounds?

Ans. 27.

11. How many miles are there in 1,585,267,200 inches?      Ans. 25020.
12. In 4000 nails, how many yards?      Ans. 250.
13. In 8000 square rods, how many acres?      Ans. 50.
14. In 2016 pints of wine, how many tuns?      Ans. 1.
15. How many bushels are there in 80,000 quarts?      Ans. 2500.
16. In 2,522,880,000 seconds, how many days?      Ans. 29,200.
17. In 3840 solid feet, how many cords?      Ans. 30.
18. In 1728 half pints of beer, how many hogsheads?      Ans. 2.
19. Bring 240,000 pence to pounds.      Ans. £1000.
20. Bring 112 quarters to cwt.      Ans. 28 cwt.
21. Bring 120 miles into leagues.      Ans. 40L.
22. Bring 1280 poles into furlongs.      Ans. 32 fur.
23. Reduce 960 nails to quarters.      Ans. 240 qrs.
24. Reduce 17280 cubick, or solid inches, to solid feet.      Ans. 10 solid feet.
25. In 768 pints, how many bushels?      Ans. 12.
26. In 1890 gallons, how many hogsheads?      Ans. 30.

- Q. 1. What does Reduction teach?
2. By what rules are its operations performed?
  3. When performed by multiplication, what is it called?
  4. What is your rule for Reduction Descending?
  5. When performed by Division what is it called?
  6. What is your rule for Reduction Ascending?
  7. How do you reduce Federal Money from a lower to a higher denomination?
  8. How is Reduction proved?

## COMPOUND ADDITION.

Compound Addition teaches to add numbers which represent articles of different value, as pounds, shillings, pence; or yards, feet, inches, &c. called *different denominations*. The operations are to be regulated by the value of the articles, which must be learned from the foregoing table.

### RULE.

Place the numbers to be added so that those of the same denomination may stand directly under each other. Add the figures of the first column or denomination together, and divide the amount by the number which it takes of this denomination to make one of the next higher. Set down the remainder, and carry the quotient to the next denomination. Find the sum of the next column or denomination, and proceed as before through the whole, until you come to the last column, which must be added by carrying one for every ten as in Simple Addition.

### EXAMPLES.

I.				II.				III.			
£	s.	d.	qrs.	£	s.	d.	qrs.	£	s.	d.	qrs.
14	10	8	2	19	19	11	3	18	17	11	3
11	16	10	1	10	14	4	1	15	14	10	3
8	3	11	3	13	13	10	2	17	18	9	2
<hr/>				<hr/>				<hr/>			
34	11	6	2	44	8	2	2	52	11	8	0Ans.
<hr/>				<hr/>				<hr/>			

In the first of the above examples, I begin with the right hand column, or that of farthings; and having added it, find that it contains 6. Now, as 6 farthings contain 1 penny and 2 over, I set the 2 farthings, under the column of farthings, and carry the penny to the column of pence. In the column of pence I find 29, which, with



the one carried from the farthings, make 30. In 30 pence I find there are 2 shillings and 6 pence over: setting the 6 pence under the column of pence, I add the 2 shillings to the column of shillings. In this column are 29, and the 2 added make 31. Thirty-one shillings contain 1 pound, and 11 shillings over. The 11 shillings are then placed under the column of shillings, and the 1 is carried to the column of pounds. In that column are 33 pounds, which, with the 1 added, make 34. Thus the amount of the sum is 34 pounds, 11 shillings, 6 pence, and 2 farthings.

In all cases in Compound Addition, one must be carried for the number of times that the higher denomination is contained in the column of the lower denomination. Thus, in Troy Weight: as 24 grains make one pennyweight, *one* from the column of grains is carried for every 24; in the column of pennyweights, one for every 20; and in every instance the learner must be guided by the foregoing table of "Money, Weights, Measures, &c."

IV.				V.			
£	s.	d.	qrs.	£	s.	d.	qrs.
487	16	11	3	9876	15	4	3
830	10	9	1	2123	14	5	0
500	11	4	2	6789	18	10	2
620	18	3	3	1234	15	11	1
900	8	10	0	7876	4	9	3

*Note.*—Sums in Compound Addition may be proved in the same manner as in Simple Addition.

## TROY WEIGHT.

VI.				VII.			
lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
487	10	18	22	6780	11	11	12
500	8	11	10	1100	9	18	22
234	11	10	16	3090	10	17	20
876	3	17	23	2468	8	13	19

## AVOIRDUPOIS WEIGHT.

VIII.					
<i>Ton.</i>	<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
16	18	2	25	11	14
97	12	3	17	9	11
34	11	1	10	10	10
82	19	2	27	15	13

IX.					
<i>Ton.</i>	<i>cwt.</i>	<i>qr.</i>	<i>lb.</i>	<i>oz.</i>	
27	17	3	27	8	
98	19	2	11	9	
70	11	1	18	7	
18	16	0	10	6	

## APOTHECARIES WEIGHT.

X.					
<i>℥</i>	<i>ʒ</i>	<i>ʒ</i>	<i>℥</i>	<i>gr.</i>	
74	9	7	1	13	
18	11	6	2	17	
91	10	3	0	10	
17	9	5	1	19	

XI.					
<i>℥</i>	<i>ʒ</i>	<i>ʒ</i>	<i>℥</i>	<i>gr.</i>	
20	10	7	1	18	
37	11	5	2	17	
28	9	3	1	15	
14	8	4	0	11	

## LONG MEASURE.

XII.					
<i>deg.</i>	<i>mil.</i>	<i>fur.</i>	<i>po.</i>	<i>ft.</i>	<i>in.</i>
118	36	7	19	13	3
921	15	4	16	10	10
671	10	6	27	11	11
643	26	5	15	8	8
123	14	5	16	7	8

XII.					
<i>mil.</i>	<i>fur.</i>	<i>po.</i>	<i>yd.</i>	<i>ft.</i>	
976	2	13	4	2	
867	6	10	3	1	
500	1	11	0	0	
123	4	15	3	2	
345	6	17	1	0	

## CUBICK, OR SOLID MEASURE.

XIV.		
<i>Ton</i>	<i>ft.</i>	<i>in.</i>
17	10	1229
24	13	1460
98	25	1527
18	16	1079

XV.		
<i>yd.</i>	<i>ft.</i>	<i>in.</i>
29	20	1092
11	11	1195
18	11	1000
27	9	1330

XVI.		
<i>Cord</i>	<i>ft.</i>	<i>in.</i>
48	120	1630
54	110	1500
75	88	1264
87	113	1128

## LAND OR SQUARE MEASURE.

XVII.

<i>acr.</i>	<i>roo.</i>	<i>per.</i>
987	2	23
798	3	28
123	2	11
567	1	27
700	0	00

XVIII.

<i>acr.</i>	<i>roo.</i>	<i>per.</i>
8423	1	36
1234	0	10
4821	3	11
6789	2	30
8000	1	13

XIX.

<i>acr.</i>	<i>roo.</i>	<i>per.</i>
9432	3	24
4324	2	12
5678	1	36
5865	3	11
8765	2	15

## CLOTH MEASURE.

XX.

<i>yd.</i>	<i>qr.</i>	<i>nl.</i>
175	3	3
481	2	1
234	1	2
345	0	1
234	1	2

XXI.

<i>El.</i>	<i>Fr.</i>	<i>qr.</i>	<i>nl.</i>
247	2	3	
456	1	1	
345	3	0	
236	2	2	
567	0	1	

XXII.

<i>El.</i>	<i>Fl.</i>	<i>qr.</i>	<i>nl.</i>
9876	2	3	
8765	1	2	
3456	2	3	
4000	0	0	
7898	2	3	

XXIII.

<i>El.</i>	<i>E.</i>	<i>qr.</i>	<i>nl.</i>
87654	1	2	
56788	3	1	
87654	3	2	
12345	0	0	
84231	2	3	

XXIV.

<i>yd.</i>	<i>qr.</i>	<i>nl.</i>
656547	1	1
987654	2	0
765432	1	3
134545	3	2
584050	0	1

XXV.

<i>yd.</i>	<i>qr.</i>	<i>nl.</i>
987654321	3	3
234567876	0	0
543212345	3	2
900087654	1	3
384563200	3	0

## DRY MEASURE.

XXVI.

<i>bush.</i>	<i>pk.</i>	<i>qt.</i>
187	7	3
290	6	2
185	3	1
349	1	2
160	5	3

XXVII.

<i>bush.</i>	<i>pk.</i>	<i>qt.</i>
356	3	7
120	1	6
543	2	1
678	3	5
432	1	3

XXVIII.

<i>bush.</i>	<i>pk.</i>	<i>qt.</i>
874	3	6
123	1	2
345	2	5
753	1	7
936	2	4

## WINE MEASURE.

XXIX.

<i>Tun.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
4820	1	16	2	1
9765	3	18	3	1
8645	2	19	1	0
5432	1	22	3	1
6787	1	10	1	0

XXX.

<i>Tun.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
987654	1	12	1	1
321234	3	15	0	1
125780	2	18	3	1
876531	2	27	1	0
248765	1	49	2	1

## ALE OR BEER MEASURE.

XXXI.

<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
4820	48	3	1
8765	34	1	1
9877	53	2	1
1234	12	1	0
5678	50	0	1

XXXII.

<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
17819174	18	3	1
21350000	27	1	1
12168400	35	0	0
21346870	15	3	1
43212345	50	1	1

## TIME.

XXXIII.

<i>w.</i>	<i>d.</i>	<i>h.</i>	<i>m.</i>	<i>s.</i>
3	6	23	58	24
3	5	20	49	57
1	4	21	30	30
3	2	13	53	53
1	0	10	10	10

XXXIV.

<i>y.</i>	<i>mo.</i>	<i>w.</i>	<i>d.</i>	<i>h.</i>	<i>m.</i>	<i>s.</i>
75	11	3	6	22	50	57
18	10	2	5	16	16	15
84	11	1	4	15	10	10
40	9	1	0	00	00	00
80	10	1	1	11	11	11

## MOTION.

XXXV.

18°	54'	44"
20	25	30
87	30	10
00	11	11
27	29	34

XXXVI.

26°	19'	15"
19	26	20
50	15	19
33	10	11
12	34	31

XXXVII.

10s.	24°	53'	50"
9	0	19	31
3	9	23	42
8	17	44	45
7	10	20	10

- Q. 1. What does Compound Addition teach?  
 2. How do you place the different denominations in Compound Addition?  
 3. How do you proceed after placing the denominations under each other?  
 4. What do you observe, in carrying from one denomination to another, that is different from Simple Addition?  
 5. How is Compound Addition proved?

## COMPOUND SUBTRACTION.

Compound Subtraction teaches to find the difference between any two sums of different denominations.

### RULE.

Place those numbers under each other which are of the same denomination—the less always being below the greater.\* Begin with the least denomination, and if it be larger than the figure over it, consider the upper one as having as many added to it as make one of the next greater denomination. Subtract the lower from the upper figure, thus increased, and set down the remainder, remembering, that whenever you thus make the upper figure larger, you must add one to the next superior denomination.

### PROOF.

As in Simple Subtraction.

### EXAMPLES.

### ENGLISH MONEY.

I.				II.							
£	s.	d.	qrs.	£	s.	d.	qrs.	£	s.	d.	qr.
460	14	10	3	744	10	8	1	689	7	9	2
320	10	8	2	398	18	10	3	372	18	4	3
<hr/>				<hr/>				<hr/>			
140	4	2	1	345	11	9	2	316	9	4	3
<hr/>				<hr/>				<hr/>			

\*By this is meant that the lower line of figures must always be a *less* sum than the upper line, though some of its smaller denominations may be larger than those immediately above them, in the upper line

The first example, is in itself, sufficiently plain. In the second, finding the upper figure smaller than the lower one, as it is in farthings, and as four farthings make a penny, I suppose four added to the upper figure, which makes it 5. Then I say, 3 from 5, and 2 remain. Placing the 2 underneath, I add 1 to the next lower figure, namely, the 10, which thus becomes 11; and as the 8 standing above is less, I suppose 12 added to it, which makes it 20. Taking 11 from 20, 9 remain. Setting the 9 underneath, and adding one to the 18, it becomes 19; and as the upper figure is smaller, I suppose 20 added to it, which makes it 30. I take 19 from 30, and 11 remain. Placing the 11 underneath, I carry one to the next figure, namely, 8; and then proceed as in Simple Subtraction.

## TROY WEIGHT.

IV.				V.			
lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
947	5	13	16	876543	7	16	11
123	10	18	20	549876	9	17	19

## AVOIRDUPOIS WEIGHT.

VI.				VII.					
Ton.	cwt.	qr.	lb.	Ton.	cwt.	qr.	lb.	oz.	dr.
5	13	1	12	8	16	0	24	11	11
1	11	3	16	6	18	2	26	12	13

## APOTHECARIES WEIGHT.

VIII.					IX.				
lb	3	3	6	gr.	lb	3	3	6	gr.
44	7	5	1	12	87	4	1	0	10
39	9	6	2	16	48	10	4	1	18

## LONG MEASURE.

X.						XI.			
<i>deg.</i>	<i>mil.</i>	<i>fur.</i>	<i>po.</i>	<i>ft.</i>	<i>in.</i>	<i>deg.</i>	<i>mil.</i>	<i>fur.</i>	<i>po.</i>
85	53	7	16	10	10	95	10	3	12
60	57	0	27	14	11	79	44	6	13

## CUBICK, OR SOLID MEASURE.

XII.			XIII.			XIV.		
<i>Ton</i>	<i>ft.</i>	<i>in.</i>	<i>yd.</i>	<i>ft.</i>	<i>in.</i>	<i>Cord</i>	<i>ft.</i>	<i>in.</i>
18	17	1040	40	10	940	874	110	1122
11	21	1485	32	16	1080	499	120	1699

## LAND, OR SQUARE MEASURE.

XV.			XVI.			XVII.		
<i>acr.</i>	<i>roo.</i>	<i>per.</i>	<i>acr.</i>	<i>roo.</i>	<i>per.</i>	<i>acr.</i>	<i>roo.</i>	<i>per.</i>
987	2	23	8423	1	36	9432	3	12
798	3	28	4123	0	10	7324	2	24

## CLOTH MEASURE.

XVIII.			XXIX.			XX.			XXI.		
<i>yd.</i>	<i>qr.</i>	<i>nl.</i>	<i>E. E.</i>	<i>qr.</i>	<i>nl.</i>	<i>E. Fl.</i>	<i>qr.</i>	<i>nl.</i>	<i>E. Fr.</i>	<i>qr.</i>	<i>nl.</i>
45	1	2	537	2	1	567	1	2	945	3	3
29	3	1	409	3	3	389	2	1	739	5	2

## DRY MEASURE.

XXII.			XXIII.			XXIV.		
<i>bush.</i>	<i>pk.</i>	<i>qt.</i>	<i>bush.</i>	<i>pk.</i>	<i>qt.</i>	<i>bush.</i>	<i>pk.</i>	<i>qt.</i>
74	1	1	230	0	0	56	1	1
42	3	2	199	2	1	28	3	3

## WINE MEASURE.

XXV.

<i>Tun.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
482	1	16	1	1
297	3	22	3	1

XXVI.

<i>Tun.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
654	2	12	1	0
276	3	40	2	1

## ALE OR BEER MEASURE.

XXVII.

<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
8240	12	1	1
1987	52	2	2

XXVIII.

<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
11917400	10	0	0
11654000	27	2	2

## TIME.

XXIX.

<i>w.</i>	<i>d.</i>	<i>h.</i>	<i>m.</i>	<i>s.</i>
8	2	12	42	30
1	1	16	54	40

XXX.

<i>y.</i>	<i>mo.</i>	<i>w.</i>	<i>d.</i>	<i>h.</i>	<i>m.</i>	<i>s.</i>
20	10	1	4	10	27	37
11	11	3	5	17	40	54

## MOTION.

XXXI.

16°	15'	35"
12	45	48

XXXII.

8s.	10°	10'	10"
6	15	50	30

XXXIII.

7s.	8°	37'	47"
4	11	44	55

*Application of the two preceding rules.*

1. A B & C purchased goods in partnership. A paid 12 pounds, 10 shillings and 8 pence; B paid 124 pounds, 16 shillings; and C paid 8<sup>7</sup> pounds and 11 pence.—What was the whole amount paid? Ans. £224 7s. 7d.

2. A merchant has money due him:—from one man, 587 pounds; from another, 420 pounds, 17 shillings and 6 pence; from a third, 200 pounds; and from a fourth, 978 pounds, 16 shillings and 8 pence. How much had he due in all? Ans. £2186 14s. 2d.



3. From 20 pounds, take 12 pounds, 19 shillings and 3 farthings. Ans. £7 0s. 11d. 1qr.

4. From 22 pounds, take 19 shillings and 1 farthing. Ans. £21 0s. 11d. 3 qrs.

5. From 17 pounds, take 9 pounds, 9 shillings and 9 pence. Ans. £7 10s. 3d.

6. A has paid B £7 2s. 3d. £19 11s. 4d. and £17 18s. 9d. on account of a debt of £30. How much remains unpaid? Ans. £15 7s. 7d.

7. A ropemaker received 3 tons, 4 cwt., 2 quarters, and 5 pounds of hemp; of which he made into cordage 2 tons, 9 cwt. and 1 quarter. How much had he left? Ans. 15cwt. 1qr. 5lbs.

- Q. 1. What does Compound Subtraction teach?  
 2. How do you set down Compound Subtraction?  
 3. What do you do when the lower denomination is larger than the one that is above it?  
 4. How is Compound Subtraction proved?

## COMPOUND MULTIPLICATION.

Compound Multiplication teaches how to find the value of any given number of different denominations, repeated a certain number of times. It is of great use in finding the value of goods, which is generally done by multiplying the price by the quantity.

### CASE I.

*When the quantity or multiplier does not exceed 12.*

Set down the price of 1, and place the multiplier under the lowest denomination; and in multiplying by it, observe the same rules for carrying from one denomination to another as in Compound Addition.

### PROOF.

Double the multiplicand, and multiply by half the multiplier: or, divide the product by the multiplier.

## EXAMPLES.

## ENGLISH MONEY.

1. What will 7 yards of cloth cost, at £1 12s. 10½d. per yard?

7

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£11 10s. 3¼d.

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In this example, I say 7 time 3 make 21—that is, 21 farthings, equal to five pence and one farthing. I set down the one farthing under the place of farthings, and carry the five pence to the place of pence saying, 7 times 10 are 70, and 5 make 75 pence—equal to 6 shillings and 3 pence. I set down the 3 pence under the pence in the sum and carry the 6 shillings, saying 7 times 12 are 84, and 6 make 90 shillings, equal to 4 pounds and 10 shillings. Setting down the 10 shillings under the shillings, I carry the 4 pounds, saying 7 times 1 are 7, and 4 make 11 pounds, making the answer to the question 11 pounds, 10 shillings, 3 pence and 1 farthing.

	II.			III.			IV.		
	£	s.	d.	£	s.	d.	£	s.	d.
Multiply	4	14	10½	7	12	9	14	15	9½
by			2			4			8
	<hr/>			<hr/>			<hr/>		
	9	9	9½						
	<hr/>			<hr/>			<hr/>		
	V.			VI.			VII.		
	£	s.	d.	£	s.	d.	£	s.	d.
Multiply	14	17	8½	24	16	10½	50	15	5½
by			9			7			12
	<hr/>			<hr/>			<hr/>		
	<hr/>			<hr/>			<hr/>		

## TROY WEIGHT.

	VIII.				IX.			
	lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
Multiply	11	9	16	10	17	5	12	6
by				4				5
	<hr/>				<hr/>			
	<hr/>				<hr/>			

AVOIRDUPOIS WEIGHT.

x.							xi.						
	Ton.	cwt.	qr.	lb.	oz.	dr.		Ton.	cwt.	qr.	lb.	oz.	dr.
Mult.	3	11	3	10	5	4		6	17	3	13	2	15
by						6							8
	<hr/>							<hr/>					
	<hr/>							<hr/>					

APOTHECARIES WEIGHT.

XII.						XII.					
	<i>℔</i>	<i>ʒ</i>	<i>ʒ</i>	<i>℥</i>	<i>gr.</i>		<i>℔</i>	<i>ʒ</i>	<i>ʒ</i>	<i>℥</i>	<i>gr.</i>
Mult.	43	10	6	2	10		4	10	7	2	13
by					5						6
	<hr/>						<hr/>				
	<hr/>						<hr/>				

LONG MEASURE.

xiv.							xv.				
	<i>deg.</i>	<i>m.</i>	<i>fur.</i>	<i>p.</i>	<i>yd.</i>	<i>ft.</i>	<i>in.</i>	<i>L.</i>	<i>m.</i>	<i>fur.</i>	<i>p.</i>
Mult.	7	22	6	20	2	2	10	15	2	7	30
by							2				6
	<hr/>							<hr/>			
	<hr/>							<hr/>			

CUBICK, OR SOLID MEASURE.

XVI.			XVII.			XVIII.		
	<i>Ton.</i>	<i>ft.</i>	<i>in.</i>		<i>yd.</i>	<i>ft.</i>	<i>in.</i>	
Mult.	10	16	15		14	2	19	
by			2				4	
	<hr/>				<hr/>			
	<hr/>				<hr/>			

LAND, OR SQUARE MEASURE.

XIX.			XX.			XXI.		
	<i>A.</i>	<i>R.</i>	<i>P.</i>		<i>A.</i>	<i>R.</i>	<i>P.</i>	
Mult.	20	3	12		37	2	15	
by			2				4	
	<hr/>				<hr/>			
	<hr/>				<hr/>			

## CLOTH MEASURE.

XXII.			XXIII.			XXIV.			XXV.					
<i>yd.</i>	<i>qr.</i>	<i>nl.</i>	<i>El.</i>	<i>Fr.</i>	<i>qr.</i>	<i>nl.</i>	<i>El.</i>	<i>Fl.</i>	<i>qr.</i>	<i>nl.</i>	<i>El.</i>	<i>E.</i>	<i>qr.</i>	<i>nl.</i>
Mult.	17	3	3	32	2	1	42	2	1	53	2	1		
by		4				6				8				9

## DRY MEASURE.

	XXVI.			XXVII.			XXVIII.		
	<i>bush.</i>	<i>pk.</i>	<i>qt.</i>	<i>bush.</i>	<i>pk.</i>	<i>qt.</i>	<i>bush.</i>	<i>pk.</i>	<i>qt.</i>
Mult.	6	3	7	14	3	2	34	2	3
by		5				6			8

## WINE MEASURE.

	XXIX.					XXX.				
	<i>Tun.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>	<i>Tun.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
Mult.	1	2	12	3	1	2	3	40	3	1
by				4						10

## ALE, OR BEER MEASURE.

	XXXI.				XXXII.			
	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>	<i>hhd.</i>	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>
Mult.	3	12	2	1	4	15	3	1
by			5				8	

## TIME.

XXXIII.					XXXIV.						
	<i>y.</i>	<i>mo.</i>	<i>w.</i>	<i>d.</i>	<i>y.</i>	<i>mo.</i>	<i>w.</i>	<i>d.</i>	<i>h.</i>	<i>m.</i>	<i>s.</i>
Mult.	7	7	3	5	8	5	3	6	20	32	10
by				9							7

## MOTION.

XXXV.			XXXVI.		
Mult.	24°	19' 11"	10s.	30°	17' 10½"
by		10			12

## CASE II.

*When the multiplier or quantity exceeds 12, and is the product of two factors in the Multiplication Table; that is, of two numbers which being multiplied together, amount to the same as the multiplier.*

Multiply the sum by one of the two numbers, and then multiply the product by the other.

## EXAMPLES.

I.				II.			
	£	s.	d.		£	s.	d.
Multiply	8	18	11½ by 18.		13	12	9½ by 27.
			6				9
	<hr/>				<hr/>		
	53	13	10½		122	15	1½
			3				3
	<hr/>				<hr/>		
	161	1	7½		368	5	4½
	<hr/>				<hr/>		

	£	s.	d.		£	s.	d.
3. Multiply	10	10	10 by 14.	Product	147	11	8
4. " "	11	11	11 by 15.	"	173	18	9
5. " "	12	12	9 by 24.	"	303	6	0
6. " "	5	13	4½ by 28.	"	158	14	6
7. " "	4	15	10 by 42.	"	201	5	0
8. " "	7	17	7½ by 64.	"	504	9	4
9. " "	6	10	3 by 72.	"	468	18	0
10. " "	9	19	11½ by 81.	"	809	16	7½
11. " "	10	15	9½ by 84.	"	906	8	3
12. " "	3	11	7½ by 96.	"	343	16	0

## CASE III.

*When the quantity, or multiplier, is such a number that no two numbers in the Multiplication Table will produce it.*

Multiply the sum by two numbers whose product will amount to nearly the same as the multiplier; then multiply the sum by the number which will make the product of the two numbers equal to the multiplier, and add its product to the sum produced by the two numbers.

## EXAMPLES.

	I.						
	£.	s.	d.		£.	s.	d.
Multiply	7	10	5		7	10	5
by 62			10				2
	<hr/>				<hr/>		
	75	4	2		15	0	10
			6		<hr/>		

451 5 0      Here note, I multiply by 10, then  
 15 0 10      by 6, because 10 times 6 make 60;  
 ——— —      then I multiply the same sum by  
 466 5 10      2, that I multiplied, first, by 10,  
 ——— —      and *add* its product to the other  
 product, which makes the amount of the answer.

	£	s.	d.		£	s.	d.
2. Multiply	2	10	10	by 31.	Product	78	15 10
3.      "	3	11	11	by 38.	"	136	12 10
4.      "	4	11	24	by 68.	"	310	0 9
5.      "	1	8	8	by 26.	"	37	5 4
6.      "	1	3	3½	by 47.	"	54	14 8½
7.      "	1	2	4½	by 83.	"	92	17 1½

## CASE IV.

*When the multiplier is greater than the product of any two numbers in the Multiplication Table.*

Multiply the sum by 10, and that product by 10, which is equal to multiplying by 100; then multiply the product by the number of hundreds in the multiplier, and if the sum be even hundreds, the product will be the answer. If there be odd numbers over even hundreds, as 70, 80, or 87, &c., multiply the amount or product of the first multiplication by 10, by the number of tens over 100; thus, if there be 70 over, multiply by 7. If, in addition to tens, there are smaller numbers, as 7, 8, 5, &c., the sum must be multiplied by such number; and the amount of all the multiplications being then added together, their sum will be the answer.

EXAMPLES.

	£	s.	d.	
Multiply	1	2	4	
by 4300			10	
	11	3	4	amount of 10.
			10	
	111	13	4	amount of 100.
			10	
	1116	13	4	amount of 1000.
			4	
	4466	13	4	amount of 4000.
	335	00	0	amount of 300.
	4801	13	4	Answer.

In the foregoing example, I first multiply by 10, three times, which gives the amount of the sum multiplied by 1000; then by 4, which gives the amount of 4000.—The sum is yet to be multiplied by 300. To do this, I take the product of the sum multiplied by 100, viz. 1117. 13s. 4d. and multiply it by 3, which gives the product of the sum by 300. The sum of these is the answer.

	£	s.	d.		£	s.	d.
2. Multiply		1	4 by 190.	Product	12	13	4
3. “	1	2	3 by 430.	“	478	7	6
4. “		7	6 by 506.	“	189	15	0
5. “		8	8 by 684.	“	296	8	0
6. “	1	3	9 by 375.	“	445	6	3
7. “		1	2 by 3456.	“	201	12	0

APPLICATION.

1. What do 84 pounds of sugar cost at 9d. per pound?

Ans. £3. 3s.

2. What do 18 yards of cloth cost at 19s per yard.

Ans. £17. 2s.

3. Sold 7 tons of iron at £32 10s. per ton; how much is the amount?

Ans. £227. 10s.

4. What is the weight of 4 hogsheads of sugar, each weighing 7 cwt. 3 qrs. 19 lb?   Ans. 31 cwt. 2 qrs. 20 lbs.
  5. What is the weight of 6 chests of tea, each weighing 3 cwt. 2 qrs. 9 lbs?       Ans. 21 cwt. 1 qr. 26 lbs.
  6. What is the value of 79 bushels of wheat, at 11s. 5½d. per bushel?               Ans. £45 6s. 10½d.
  7. What is the value of 94 barrels of cider, at 12s. 2d. per barrel?               Ans. £57 3s. 8d.
  8. What is the value of 114 yards of cloth, at 15s. 3½d. per yard?               Ans. £87 5s. 7½d.
  9. What is the value of 12 cwt. of sugar, at £3 7s. 4d. per cwt.?               Ans. £40 8s.
  10. What is the worth of 63 gallons of oil at 2s. 3d. per gallon?               Ans. £7 1s. 9d.
  11. What is the amount of 120 days wages at 5s. 9d. per day?               Ans. £34 10s.
  12. What is the worth of 144 reams of paper at 13s. 4d. per ream?               Ans. £96.
  13. What will 1 cwt. of sugar cost, at 10½d. per lb? \*               Ans. £5 0s. 4d.
  14. If I have 9 fields, each containing 12 acres, 2 roods, and 25 poles; how many acres have I in the whole?               Ans. 113A. 3R. 25P.
  15. What will 1 ton of lead cost, at 1 pence per pound?               Ans. £37 6s. 8d.
  16. If a man can travel 25 ms. 3 fur. 20 ps. 4 yds. in 1 day, how far can he travel in 6 days?               Ans. 152ms. 5fur. 4ps. 2yds.
- Q. 1. What does Compound Multiplication teach?
2. In what is it particularly useful?
  3. Which is made the multiplier—the price, or the quantity?
  4. How do you proceed when the multiplier does not exceed 12?
  5. How do you proceed when the multiplier exceeds 12?
  6. When the multiplier consists of no two component numbers, as in case third, how do you proceed?
  7. How do you proceed in case fourth?
  8. How is compound multiplication proved?

\*It must be recollected that 1cwt. is 112lbs.



# COMPOUND DIVISION.

Compound Division teaches the manner of dividing numbers of different denominations.

## CASE I.

*When the divisor does not exceed 12.*

Begin at the highest denomination, and after dividing that, if any thing remain, reduce it to the next lower denomination, adding it to that denomination in the sum, and proceed in this manner until the whole is divided. If the number of either denomination should be too small to contain the divisor, reduce it to the next lower denomination, and add it thereto, as directed in case of a remainder. The denominations in the quotient must be kept separate.

## PROOF.

Multiply the quotient by the divisor, and the product, if right, will be equal to the dividend.

## EXAMPLES.

I.			II.			III.		
£	s.	d.	£	s.	d.	£	s.	d.
Divide 2)	6	8	4)	3	3	5)	7	2
	3	4		15	11½		1	8
		4						5½+3
IV.			V.			VI.		
£	s.	d.	£	s.	d.	£	s.	d.
5)	6	17	6)	9	9	12)	21	16
	1	7		1	11		1	16
		7			7½			4½+10

In doing the 6th sum, which is divided by 12, I find the divisor is contained once in 21; and setting down 1 I find 9 pounds remaining; which, reduced to shillings, and added to the 16 shillings in the sum, make 196 shillings. The divisor being contained 16 times in 196, with 4 remaining, I set down 16, and reducing the 4 shillings to pence, and adding them to the 11 pence, in the sum, the amount is 59 pence. The divisor is con-

tained 4 times in 59, leaving 11 pence remaining. I set down 4, and the remaining 11 pence reduced to farthings and added to the half penny or 2 farthings in the sum, make 46 farthings; and as the divisor is contained 3 times in 46, leaving a remainder of 10, I set down  $\frac{1}{4}$  and place the final remainder at the right hand of the sum.

	£	s.	d.			£	s.	d.
7. Divide	12	10	10	by 5.	Quotient	2	10	2
8. "	13	13	9	by 4.	"	3	8	5 $\frac{1}{4}$
9. "	2	18	11 $\frac{1}{2}$	by 3.	"		19	7 $\frac{1}{2}$ +1
10. "	7	7	7	by 4.	"	1	16	10 $\frac{1}{2}$
11. "	177	19	11 $\frac{1}{2}$	by 12.	"	14	16	7 $\frac{1}{2}$ +11

## CASE II.

*When the divisor exceeds 12, and is the product of two numbers multiplied together.*

Divide by one of the numbers: then divide the quotient by the other.

## EXAMPLES.

Divide £5 10s. 6d. by 48.

$$\begin{array}{r}
 \text{I.} \\
 \text{£} \quad \text{s.} \quad \text{d.} \\
 6 \overline{) 5 \quad 10 \quad 6} \\
 \hline
 8 \overline{) 18 \quad 5} \\
 \hline
 \end{array}$$

2 3 $\frac{1}{2}$  5 Answer.

*Note.*—If there be any remainder in the first operation, and not in the second, it is the true one. When there is a remainder in the second operation, multiply it by the first divisor, and add it to the first remainder, if there be any, and it forms the true remainder.

	£	s.	d.			£	s.	d.
2. Divide	240	12	10	by 16.	Quotient	15	0	9 $\frac{1}{2}$ +8
3. "	88	13	11	by 21.	"	4	4	5 $\frac{1}{2}$ +14
4. "	90	15	4 $\frac{1}{2}$	by 32.	"	2	16	8 $\frac{1}{2}$ +2
5. "	450	8	8	by 42.	"	10	14	5 $\frac{1}{2}$ +26
6. "	789	19	9	by 64.	"	12	6	10 $\frac{1}{2}$ +52
7. "	840	4	3 $\frac{1}{2}$	by 72.	"	11	13	4 $\frac{1}{2}$ +62

## CASE III.

*When the divisor is more than 12, and cannot be produced by multiplying any two numbers together.*

Divide after the manner of Long Division, reducing from higher to lower denominations, as in the following

## EXAMPLES.

Divide £61 12s. by 23.

I.

£ s.  
23)61 12 (£2 13s. 6d. 3qrs. + 3 Ans.  
46

15

20×

312

23

82

69

13

12×

156

138

18

4×

72

69

3

Divide £14 10s. 11½d. by 95.

II.

£ s. d.  
95)14 10 11½ (£0 3s. 0½d. + 2 Ans.

20×

290

285

5

12×

71

4×

28

28

2

*Note.*—In the second example, I find the divisor greater than the number of pounds in the dividend. I therefore set down a cypher in the place of pounds in the quotient, then reduce the 14 pounds in the sum, into shillings, at the same time adding the ten shillings in the

sum, which thereby becomes 290. In 290 the divisor is contained 3 times, and 5 over. This 5 shillings I reduce to pence, adding to it the 11 pence in the sum; and the amount being still smaller than the divisor, I set down a cypher in the place of pence, in the quotient; and reduce it to farthings, and proceed as before.

Though this operation is longer, it is, perhaps, less liable to error than either of the preceding cases.

	£	s.	d.			£	s.	d.
3. Divide	20	10	8	by 17.	Quotient	1	4	$1\frac{3}{4}+9$
4. " "	27	18	7	by 29.	"	0	19	$3\frac{1}{2}+4$
5. " "	147	4	4	by 65.	"	2	5	$3\frac{1}{2}+18$
6. " "	581	19	$11\frac{1}{8}$	by 73.	"	7	19	$5\frac{1}{4}+49$
7. " "	77	3	$3\frac{3}{4}$	by 19.	"	4	1	$2\frac{1}{2}+17$
8. " "	319	7	$10\frac{1}{2}$	by 29.	"	11	0	$3\frac{1}{4}+1$

#### APPLICATION.

1. If 42 cows cost £126 16s. 6d; what was the price of each? Ans. £3 0s.  $4\frac{1}{2}$ d.

3. Five men bought a quantity of hay, weighing 21 tons, 13 hundred, and 3 quarters; which they divided, equally among them. What was the share of each?

Ans. 4 tons, 6cwt 3qrs.

4. A farmer had 3 sons, to which he gave a tract of land containing 520 acres, 3 roods, 29 perches; and the land was to be divided equally among them. What was the portion of each?

Ans. 173A. 2R. 23P.

5. Divide 375 miles, 2 furlongs, 7 poles, 2 yards, 1 foot, 2 inches, by 39. Ans. 9M. 4fur. 39P. 0yd. 2ft. 8in.

Q. 1. What does Compound Division teach?

2. How do you proceed when the divisor does not exceed 12?

3. How? When the number of either denomination is too small to contain the divisor?

4. How? When the divisor exceeds 12, and is the product of two numbers multiplied together?

5. How? When the divisor is more than 12, and cannot be produced by multiplying any two numbers together?

6. How is Compound Division proved?

## EXCHANGE.

Exchange teaches to change a sum of one kind of money to a given denomination of another kind.

*To reduce the currency of each of the United States to dollars and cents, or Federal Money.*

### RULE.

Reduce the sum to pence; to the pence annex two cyphers; then divide by the number of pence in a dollar, as it passes in each State, the quotient or answer will be in cents, which may be easily reduced to dollars.

*Note.*—This rule applies to the currency of any other country, if its currency be in pounds, shillings, pence, &c.

### EXAMPLES.

1. Reduce 621 pounds, New England Virginia, and Kentucky currency, to dollars and cents; a dollar being 72 pence.

$$\begin{array}{r}
 \text{£} \\
 621 \\
 20 \\
 \hline
 12420 \\
 12 \\
 \hline
 72 \overline{)14904000} (\$2070.00 \\
 \underline{144} \\
 504 \\
 \underline{504} \\
 000
 \end{array}$$

2. Reduce 12 pounds, 3 shillings, and 9 pence to dollars and cents.

Ans. \$40.62½.

3. Reduce 30 pounds and 3 shillings to dollars and cents.

Ans. \$100.50.

4. In £763 how many dollars, cents and mills?

Ans. \$2543.33cts. 3 m.

5. Reduce 19 shillings and 10 pence to dollars and cents.

Ans. \$3.30cts. 5m.

6. In 9 pounds and 16 shillings in New York and North Carolina currency, how many dollars and cents, reckoning 96 pence to a dollar? Ans. \$24.50.

7. In 30 pounds, how many dollars and cents, same currency? Ans. 75.00.

8. In 27 pounds, 2 shillings, how many dollars and cents, same currency? Ans. 67.75.

9. In 942 pounds of New Jersey, Pennsylvania, Delaware, and Maryland currency; how many dollars and cents, a dollar being 90 pence? Ans. \$2512.00.

10. In 12 pounds how many dollars and cents same currency? Ans. \$32.00.

11. In 86 6s. 5d. how many dollars, cents and mills, same currency? Ans. \$230.18 cts. 8m

12. In 21 pounds, South Carolina and Georgia currency, how many dollars and cents, there being 56 pence in a dollar? Ans. \$90.00.

13. In 56 pounds, how many dollars, &c. same currency? Ans. \$240.00.

14. 108 pounds, Canada and Nova Scotia currency, how many dollars, &c. there being 60 pence in a dollar? Ans. \$432.00.

15. In 460 pounds and 16 shillings sterling, or English money, how many dollars, &c. there being 54 pence in a dollar? Ans. \$2048.00.

16. Reduce 16 pounds, 6 shillings, and 3 pence English money, to dollars and cents. Ans. \$72.50.

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*To bring Federal Money into pounds shillings, & pence.*

#### RULE.

Multiply the dollars, or dollars and cents, by the number of pence in a dollar of the currency to which you wish to change the given sum; the answer will be in pence, which can then be reduced to shillings and pounds. When there are cents in the sum to be reduced, two figures must be cut off from the right of the product, before bringing it into pounds.

*Note.*—This rule applies to the currency of any country whose currency is in pounds, shillings, &c.

## EXAMPLES.

1. In \$16.50 how many pounds and shillings, in sterling, or English money; a dollar being four shillings and six pence, or 54 pence?

$$9 \times 6 = 54$$

\$16.50

9

---

148.50

6

---

12)891.00

---

2|0)7|4.+3

£3 14s. 3d. Answer.

2. In 33 dollars, how many pounds, &c.?

Ans. £7. 8s.—6d.

3. In 1000000 dollars, how many pounds sterling?

Ans. £225000.\*

4. Reduce 432 dollars into the currency of Canada and Nova Scotia, a dollar being equal to five shillings; or 60 pence.

Ans. £108.

5. In \$490.50 how many pounds, shillings, &c. same currency?

Ans. £122 12s. 6d.

6. Bring \$150.25, into the currency of New England, Virginia, and Kentucky, a dollar, being equal to 72 pence.

Ans. £45. 1s. 6d.

*Questions.*

1. What does Exchange teach?
2. How do you reduce the currency of any one of the United States to Federal Money?
3. Does this rule apply to the currency of any other country?
4. How do you change Federal Money into pounds, shillings, and pence of any state or country?
5. Among the various kinds of money, what kind is the most easily reckoned?

\*Federal Money may, also, be changed into English Money, by multiplying the dollars by 9, and dividing the product by 40.







What part of eight are two? What part of eight are six? What part of 9 are 6? What part of 10 are 2? What part of 10 are 4? What part of 10 are 7? What part of 12 are 6? What part of 12 are 4? What part of 12 are 3? What part of 12 are 2?

10. How many are two fourths of 12? How many are three fourths of 12? Two thirds of 12, are how many? How many are 5 times 8? In one eighth of 40, how many? In three eighths of 40, how many? Four eighths of 40, are how many? Then  $\frac{1}{4}$  of any number, or of any thing, amount to how many, or how much? How many are  $\frac{2}{3}$  of 30? How many are  $\frac{1}{3}$  of 30? How many in  $\frac{1}{2}$  of 60? In  $\frac{1}{4}$  of 60, how many? In  $\frac{1}{3}$  of 60, how many? In  $\frac{1}{6}$  of 60, how many? How many are  $\frac{5}{6}$  of 60? How many are  $\frac{2}{3}$  of 60? How many are  $\frac{5}{6}$  of 60? In 2 and  $\frac{1}{5}$ , how many fifths? In 5 and  $\frac{4}{5}$  how many fifths? In  $\frac{1}{5}$  of 100, how many? In  $\frac{1}{4}$  of 100 cents, or 1 dollar, how much? How much are  $\frac{1}{2}$  and  $\frac{1}{4}$ ? How much are  $\frac{1}{2}$  and  $\frac{2}{4}$ ? How much are  $\frac{1}{3}$  and  $\frac{2}{3}$ ? How much are  $\frac{2}{3}$ ? How much are  $\frac{3}{4}$  and  $\frac{1}{4}$ ? How much are  $\frac{2}{5}$ ,  $\frac{3}{5}$ , and  $\frac{1}{5}$ ? In  $1\frac{1}{4}$ , how many? In  $\frac{7}{8}$ , how many? If you take  $\frac{1}{2}$  from one dollar, how much will remain? If you take  $\frac{2}{3}$  from one dollar, how much will remain? If you take  $\frac{1}{5}$  from a pound, how much will remain? If you take  $\frac{5}{8}$  from one, how much will remain? How many fourths are 2 times  $\frac{1}{4}$ ? How many are 5 times  $\frac{2}{3}$ ? How many are 3 times  $\frac{1}{3}$ ? In  $1\frac{1}{4}$ , how many? In  $1\frac{3}{4}$ , how many?

11. If one half, three fourths and a quarter, be added, how much will be their amount?

12. If you take two eighths from eleven eighths, how much will remain?

13. What is a proper fraction?

Ans. When the numerator is less than the denominator, as  $\frac{1}{2}$ , or  $\frac{2}{3}$ , &c.

14. What is an improper fraction?

Ans. It is that in which the numerator is equal, or superior to the denominator; as,  $\frac{3}{3}$ , or  $\frac{4}{4}$ , or  $\frac{7}{7}$ , &c.

15. What is a simple fraction?

Ans. It is a fraction expressed in a simple form; as,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{5}{8}$ .

16. What is a compound fraction?

Ans. It is the fraction of a fraction, or several fractions connected together with the word *of* between them; as  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{3}{4}$ ; or  $\frac{1}{2}$  of  $\frac{7}{12}$ , &c. which are read thus, one half of two thirds; &c.

17. What is a mixed number?

Ans. It is composed of a whole number and a fraction; as  $3\frac{1}{2}$ , or  $12\frac{1}{2}$ .

18. What is the common measure of two or more numbers?

Ans. It is that number which will divide each of them without a remainder; thus 5 is the common measure of 10, 20, and 30; and the greatest number that will do this, is called the greatest common measure.

19. What is meant by the common multiple?

Ans. Any number which can be measured by two or more numbers, is called the common multiple of those numbers; and if it be the least number that can be so measured, it is called the least common multiple; thus 40, 60, 80, 100; are multiples of 4 and 5; but their least common multiple is 20.

20. When is a fraction said to be in its lowest terms?

Ans. When it is expressed by the smallest numbers possible.

21. What is meant by a prime number?

Ans. It is a number which can only be measured by itself, or an unit.

22. What is meant by a composite number?

Ans. That number, which is produced by multiplying several numbers together, is called a composite number.

23. What is a perfect number?

Ans. A perfect number is one that is equal to the sum of its aliquot parts.\*

\*The following perfect numbers are all which are, at present known.

6	8589869056
28	137438691328
496	3205843008139952128
8128	24178521639228158637784575
33550336	9903520314282971830448816128

## REDUCTION OF VULGAR FRACTIONS.

Reduction of Vulgar Fractions, is the bringing of them out of one form into another, in order to prepare them for Addition, Subtraction, Multiplication, &c.

## CASE I.

*To reduce a fraction to its lowest terms.*

## RULE.

Divide the greater term by the less, and that divisor by the remainder, and so continue till nothing be left; the last divisor will be the *common measure*; then divide both parts of the fraction by the common measure, and the quotients will express the fraction required.

*Note.*—If the common measure happen to be 1, the fraction is already at its lowest term. Cyphers, on the right hand side of both terms, may be rejected; as  $\frac{9}{10}$ ,  $\frac{5}{7}$ .

## EXAMPLES.

1. Reduce  $\frac{144}{240}$  to its lowest terms.

$$\begin{array}{r} 144 \overline{)240}(1 \\ \underline{144} \end{array}$$

$$\begin{array}{r} 48 \overline{)144}(3 \\ \underline{144} \end{array}$$

$$\begin{array}{r} 96 \overline{)144}(1 \\ \underline{96} \end{array}$$

$\frac{3}{5}$  Ans.

Greatest common —

$$\begin{array}{r} 48 \overline{)240}(5 \\ \underline{240} \end{array}$$

$$\begin{array}{r} 48 \overline{)36}(2 \\ \underline{96} \end{array}$$

Thus 48 is the greatest common measure, and the true answer is obtained by dividing the fraction by it.

This reduction may be performed, also, by another rule, thus:—Divide the numerator and denominator of the fraction by any number that will divide them both without a remainder; divide the quotients in the same manner, and so on, till no number will divide them both, and the last quotients will express the fraction in its lowest terms.

The same sum done by this method:—

$$12 \overline{)144}(12 \quad 4 \overline{)240}(6 \text{ Answer.}$$

2. Reduce  $\frac{9}{27}$  to its lowest terms. Ans.  $\frac{1}{3}$ .

3. Reduce  $\frac{132}{576}$  to its lowest terms. Ans.  $\frac{11}{48}$ .

4. Reduce  $\frac{252}{364}$  to its lowest terms. Ans.  $\frac{9}{13}$ .

5. Reduce  $\frac{1344}{1536}$  to its lowest terms. Ans.  $\frac{7}{8}$ .

## CASE II.

*To reduce a mixed number to an improper fraction.*

## RULE.

Multiply the whole number by the denominator of the fraction, and add the numerator to the product; then set that sum, namely, the whole product, above the denominator for the fraction required.

## EXAMPLES.

Reduce  $23\frac{1}{5}$  to an improper fraction.

5

117

 $1\frac{1}{5}$  Answer.

2. Reduce  $12\frac{1}{4}$  to an improper fraction. Ans.  $1\frac{1}{4}$

3. Reduce  $14\frac{1}{10}$  to an improper fraction. Ans.  $1\frac{1}{10}$

4. Reduce  $183\frac{5}{11}$  to an improper fraction. Ans.  $2\frac{5}{11}$

## CASE III.

*To reduce an improper fraction to a whole or mixed number.*

## RULE.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number sought.

## EXAMPLES.

1. Reduce  $\frac{12}{3}$  to its equivalent number.

3)12(4 Answer.

12

2. Reduce  $\frac{15}{7}$  to its equivalent number.

7)15(2 $\frac{1}{7}$  Answer.

14

1

3. Reduce  $\frac{44}{1}$  to its equivalent number. Ans.  $44\frac{1}{1}$

5. Reduce  $\frac{8}{1}$  to its equivalent fraction. Ans. 8.

5. Reduce  $\frac{541}{1}$  to its equivalent fraction. Ans.  $541\frac{1}{1}$

6. Reduce  $\frac{171}{1}$  to its equivalent number. Ans.  $171\frac{1}{1}$

## CASE IV.

*To reduce a whole number to an equivalent fraction, having a given denominator.*

## RULE.

Multiply the whole number by the given denomina-

tor; then set down the product above for a numerator, and the given denominator below, and they will form the fraction required.

## EXAMPLES.

1. Reduce 9 to a fraction whose denominator shall be 7.  $9 \times 7 = 63$ , then  $\frac{63}{7}$  is the answer.

2. Reduce 13 to a fraction whose denominator shall be 12. Ans.  $\frac{156}{12}$ .

3. Reduce 27 to a fraction whose denominator shall be 11. Ans.  $\frac{297}{11}$ .

## CASE V.

*To reduce a compound fraction to a simple one.*

## RULE.

Multiply all the numerators together, for a new numerator, and all the denominators for a new denominator, then reduce the fraction to its lowest term.

## EXAMPLES.

1. Reduce  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $\frac{1}{2}$  to a single or simple fraction.

$$\begin{array}{r} 1 \times 2 \times 3 \quad 6 \quad 1 \\ - \quad - \quad - = - = - \text{ Answer.} \\ 2 \times 3 \times 4 \quad 24 \quad 4 \end{array}$$

2. Reduce  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{1}{2}$  to a single fraction. Ans.  $\frac{1}{4}$ .

3. Reduce  $\frac{3}{4}$  of  $\frac{4}{5}$  to a single fraction. Ans.  $\frac{12}{5}$ .

4. Reduce  $\frac{5}{6}$  of  $\frac{4}{5}$  of  $\frac{1}{2}$  to a simple fraction. Ans.  $\frac{55}{18}$ .

## CASE VI.

*To reduce fractions of different denominations to others of the same value, and having a common denominator.*

## RULE.

Multiply each numerator into all the denominators except its own, for a new numerator, and all the denominators into each other for a common denominator.\*

\*The least common denominator, or multiple, of two or more numbers, may be found thus: Divide the given denominations by any number that will divide two or more of them without a remainder, and set the quotients and undivided numbers underneath. Divide these quotients by any number that will divide two or more of them as before, and thus continue, till no two numbers are left, capable of being lessened. Then multiply the last quotients and the divisor, or divisors together, and the product will be the answer.

What is the least common multiple of  $\frac{5}{8}$ ,  $\frac{7}{15}$ , and  $\frac{6}{16}$ ?

$$\begin{array}{r} 8 \overline{) 9 \quad 8 \quad 15 \quad 16} \\ \hline \end{array}$$

$$\begin{array}{r} 3 \overline{) 9 \quad 1 \quad 15 \quad 2} \\ \hline \end{array}$$

$$\begin{array}{r} 3 \quad 1 \quad 5 \quad 2 \\ \hline \end{array}$$

$$3 \times 1 \times 5 \times 2 = 30 \times 3 \times 8 \times 720, \text{ Ans.}$$

## EXAMPLES.

1. Reduce
- $\frac{1}{2}$
- ,
- $\frac{2}{3}$
- , and
- $\frac{3}{4}$
- to a common denominator.

$$1 \times 3 \times 4 = 12 \text{ the numerator for } \frac{1}{2}.$$

$$2 \times 2 \times 4 = 16 \text{ the numerator for } \frac{2}{3}.$$

$$3 \times 2 \times 3 = 18 \text{ the numerator for } \frac{3}{4}.$$

Denominator  $2 \times 3 \times 4 = 24$  the common denominator.Therefore the results are  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4} = \frac{12}{24}$ ,  $\frac{16}{24}$ ,  $\frac{18}{24}$ .Or the multiplications may be performed mentally, and the results given  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4} = \frac{12}{24}$ ,  $\frac{16}{24}$ ,  $\frac{18}{24}$ .

2. Reduce
- $\frac{2}{7}$
- and
- $\frac{5}{9}$
- to a common denominator.

$$\text{Ans. } \frac{18}{63} \text{ and } \frac{35}{63}.$$

3. Reduce
- $\frac{2}{3}$
- ,
- $\frac{3}{4}$
- , and
- $\frac{4}{5}$
- to a common denominator.

$$\text{Ans. } \frac{40}{60}, \frac{36}{60}, \frac{48}{60}.$$

5. Reduce
- $\frac{1}{2}$
- ,
- $\frac{2}{3}$
- , and
- $\frac{5}{6}$
- to fractions of a common denominator

$$\text{Ans. } \frac{40}{120}, \frac{64}{120} \text{ and } \frac{100}{120}.$$

## CASE VII.

*To reduce the fraction of one denomination to the fraction of another, but greater, retaining the same value.*

## RULE.

Make the fraction a compound one, by comparing it with all the denominations between it and that denomination to which you would reduce it; then reduce that compound fraction to a simple one.

## EXAMPLES.

1. Reduce
- $\frac{4}{7}$
- of a cent to the fraction of a dollar. By comparing it, it becomes
- $\frac{4}{7}$
- of
- $\frac{1}{10}$
- of
- $\frac{1}{10}$
- which being reduced by case five, will be
- $4 \times 1 \times 1 = 4$
- and
- $7 \times 10 \times 10 = 700$
- .

$$\text{Ans. } \frac{4}{700} \text{ D.}$$

2. Reduce
- $\frac{2}{3}$
- of a mill to the fraction of an eagle.

$$\text{Ans. } \frac{2}{30000}.$$

3. Reduce
- $\frac{3}{5}$
- of a penny to the fraction of a pound.

$$3 \times 1 \times 1 = 3 \quad 1$$

$$\frac{3}{5} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20} \quad - \quad - \quad - \quad - = \text{£} \quad \text{Ans.}$$

$$5 \times 12 \times 20 = 1200 \quad 400$$

4. Reduce
- $\frac{4}{7}$
- of an ounce to the fraction of a pound, Avoirdupois Weight.

$$\text{Ans. } \frac{4}{7} \text{ lb.}$$

5. Reduce
- $\frac{7}{8}$
- of a dwt. to the fraction of a pound, Trow Weight.

$$\text{Ans. } \frac{7}{160} \text{ lb.}$$

6. Reduce
- $\frac{1}{11}$
- of a minute to the fraction of a day.

$$\text{Ans. } \frac{1}{1584} \text{ day.}$$

## CASE VIII.

*To reduce the fraction of one denomination to the fraction of another, but less, retaining the same value.*

## RULE.

Multiply the given numerator by the parts in the denomination between it and that to which you would reduce it, and place the product over the given denominator.

## EXAMPLES.

1. Reduce  $\frac{1}{175}$  of a dollar to the fraction of a cent. The fraction is  $\frac{1}{175}$  of  $\frac{1}{10}$  of  $\frac{1}{10}$ ; then

$$\frac{1 \times 10 \times 10}{175 \times 1 \times 1} = \frac{100}{175} \text{ and this reduced, is equal to } \text{Ans. } \frac{4}{7} \text{c.}$$

2. Reduce  $\frac{2}{30000}$  of an eagle to the fraction of a mill. Ans.  $\frac{2}{15000}$ .

3. Reduce  $\frac{1}{400}$  of a pound to the fraction of a penny. Ans.  $\frac{2}{800}$ .

4. Reduce  $\frac{1}{28}$  of a pound Avoirdupois, to the fraction of an ounce. Ans.  $\frac{4}{7}$ .

5. Reduce  $\frac{7}{1536}$  of a pound Troy, to the fraction of a pennyweight. Ans.  $\frac{7}{192}$  dwt.

6. Reduce  $\frac{1}{1440}$  of a day to the fraction of a minute. Ans.  $\frac{1}{24}$  of a min.

## CASE IX.

*To find the value of the fraction in the known parts of the integer; or, to reduce a fraction to its proper value.*

## RULE.

Multiply the numerator by the known parts of the integer, and divide by the denominator.

## EXAMPLES.

1. What is the value of  $\frac{2}{3}$  of a pound?

2 thirds of a pound.

20

$3 \overline{)40}$  thirds of a shilling.

13 s. + 1 third of a shilling.

12

$3 \overline{)12}$  thirds of a penny.

4d

Ans. 13s. 4d.



2. Reduce  $\frac{2}{5}$  of a shilling to its proper value.

2 fifths of a shilling.

12

5)24(4d.

20

4 fifths of a penny.

4

5)16 fifths of a farthing.

3 qr. + 1 fifth.

Ans. 4d. 3qr.  $\frac{1}{8}$ .

3. Reduce  $\frac{2}{5}$  of a lb. Troy, to its proper quantity.

Ans. 7 oz. 4dwt.

4. Reduce  $\frac{4}{5}$  of a mile to its proper quantity.

Ans. 6 fur. 16 poles.

5. Reduce  $\frac{5}{16}$  of a cwt. to its proper quantity.

Ans. 2 qrs.

6. Reduce  $\frac{5}{8}$  of an acre to its proper value.

Ans. 2R. 20P.

7. Reduce  $\frac{3}{5}$  of a day to its proper value.

Ans. 7 hours 12 min.

### CASE X.

*To reduce any given quantity to the fraction of a greater denomination of the same kind.*

### RULE.

Reduce the given quantity to the lowest denomination mentioned for a numerator, and the integer into the same denomination for a denominator.

### EXAMPLES.

1. Reduce 16s. 8d. to the fraction of a pound.

16 8

Integer £1

12

20

Numerator

200

20

— =  $\frac{5}{8}$  Ans.

12

Denominator 240

240 Denominator.

2. Reduce 6 furlongs and 16 poles to the fraction of a mile  
Ans.  $\frac{4}{5}$ .
3. Reduce  $\frac{2}{3}$  of a farthing to the fraction of a pound.  
Ans.  $\frac{1}{1440}$ .
4. Reduce  $\frac{4}{5}$  dwt. to the fraction of a pound Troy.  
Ans.  $\frac{1}{300}$ .
5. Bring 80 cents to the fraction of a dollar.  
A dollar is 100 cents, then 80 cents are equal to  $\frac{80}{100}$  of a dollar; which, being reduced, is equal to  $\frac{4}{5}$  Ans.
6. Bring 16 cents 9 mills to the fraction of an eagle.  
16 cents 9 mills = 169  
1 eagle =  $\frac{169}{10000}$  Ans.
7. Bring 2 quarters  $3\frac{1}{9}$  nails to the fraction of an ell English.\*

2 quarters  $3\frac{1}{9}$  nails.

4

—

11

9

—

Numerator 100

—

Denominator 9 of  $\frac{1}{4}$  of  $\frac{1}{3} = \frac{100}{180} = \frac{5}{9}$  Ans.

## ADDITION OF VULGAR FRACTIONS.

### CASE I.

*To add fractions having a common denominator.*

#### RULE.

Add all the numerators together, and place the sum over the common denominator, which will give the sum of the fractions required.

#### EXAMPLES.

1. Add  $\frac{3}{5}$ ,  $\frac{4}{5}$  and  $\frac{2}{5}$  together.

$$\frac{3}{5} \times \frac{4}{5} \times \frac{2}{5} = \frac{9}{5} = 1\frac{4}{5} \text{ Answer.}$$

\*When the sum contains a fraction, as in the seventh example, multiply both parts of the sum by the denominator thereof, and to the numerator add the numerator of the given fraction.

2. Add  $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}$  and  $\frac{5}{7}$  together.

$$\frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{5}{7} = \frac{11}{7} = 1\frac{4}{7} \text{ Answer.}$$

### CASE II.

*To add fractions having different denominators.*

#### RULE.

Find the common denominator by Case VI. in Reduction; then add, as in the preceding examples.

#### EXAMPLES.

1. Add  $\frac{3}{4}$  and  $\frac{5}{9}$  together.

$$\left. \begin{array}{l} 4 \times 5 = 20 \\ 3 \times 9 = 27 \end{array} \right\} \text{numerators.}$$

—  
47 sum.

$$4 \times 9 = 36 \text{ com. denom. } \frac{47}{36} = 1\frac{11}{36} \text{ Ans.}$$

2. Add  $\frac{2}{7}$  and  $\frac{5}{14}$  together.

$$\text{Ans. } \frac{9}{14}.$$

### CASE III.

*To add mixed numbers.*

#### RULE.

Add the fractions as in Case I, in Addition, and the whole numbers as in *Simple Addition*; then add the fractions to the sum of the whole numbers. If the fractions have different denominators, reduce them to a common denominator, and then add the fractions to the integers or whole numbers.

#### EXAMPLES.

1. Add  $13\frac{1}{5}, 9\frac{4}{5}, 3\frac{7}{5}$  together.

$$13 + 9 + 3 = 25 \text{ whole numbers.}$$

$$\frac{1}{5} + \frac{4}{5} + \frac{7}{5} = \frac{12}{5} = \frac{2}{5}. \text{ Thus, } 25\frac{2}{5} \text{ Ans.}$$

2. Add  $5\frac{2}{3}, 6\frac{7}{8}$  and  $4\frac{1}{2}$  together.

$$5 + 6 + 4 = 15 \text{ whole numbers.}$$

$$\text{Then, } 2 \times 8 \times 2 = 32$$

$$7 \times 3 \times 2 = 42$$

$$1 \times 3 \times 8 = 24$$

—  
98 sum of the numerators.

$$3 \times 8 \times 2 = 48 \text{ common denominator.}$$

$$\text{Then, } \frac{98}{48} = 2\frac{1}{24}.$$

$$\text{Thus, } 15 + 2\frac{1}{24} = 17\frac{1}{24} \text{ Answer.}$$

3. Add  $1\frac{2}{3}, 2\frac{3}{5}$  and  $3\frac{5}{7}$  together.

$$\text{Ans. } 7\frac{103}{105}.$$

## CASE IV.

*To add compound fractions.*

## RULE.

Reduce them to simple ones, and proceed as before.

## EXAMPLES.

1. Add  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{3}{4}$ , to  $\frac{2}{3}$  of  $\frac{3}{5}$  of  $\frac{1}{11}$ .

$$\begin{cases} 1 \times 2 \times 3 = 6 \\ 2 \times 3 \times 4 = 24 \end{cases} = \frac{6}{24} = \frac{1}{4} \text{ simple fraction.}$$

$$\text{and } \begin{cases} 2 \times 3 \times 10 = 60 \\ 3 \times 5 \times 11 = 165 \end{cases} = \frac{60}{165} = \frac{4}{11} \text{ simple fraction.}$$

Then find a common denominator.

for  $\frac{1}{4}$ ,  $\frac{4}{11}$  thus  $\begin{matrix} 4 \times 4 = 16 \\ 1 \times 11 = 11 \end{matrix}$  numerator.

—  
27 sum of the numerators.

—  
 $4 \times 11 = 44$  common denominator.

Therefore  $\frac{27}{44}$  is the answer.

2. Add  $\frac{2}{4}$  of  $\frac{7}{8}$ , and  $\frac{4}{6}$  of  $\frac{1}{2}$  together. Ans.  $\frac{37}{24}$ .

3. Add  $\frac{5}{6}$  of  $\frac{2}{10}$ , and  $\frac{7}{12}$  of  $\frac{4}{5}$  together. Ans.  $1\frac{11}{60}$ .

4. Add  $\frac{3}{7}$ ,  $9\frac{1}{5}$  and  $\frac{2}{3}$  of  $\frac{1}{2}$  together.

*Note.*—The mixed number of  $9\frac{1}{5} = 4\frac{6}{5}$ ; the compound fraction  $\frac{2}{3}$  of  $\frac{1}{2} = \frac{2}{6}$ . Then the fractions are,  $\frac{3}{7}$ ,  $4\frac{6}{5}$  and  $\frac{2}{6}$ ; which must be reduced to the fractions of a common denominator and added. Ans.  $9\frac{101}{55}$ .

5. Add  $1\frac{2}{10}$ ,  $6\frac{7}{8}$ ,  $\frac{2}{3}$  of  $\frac{1}{2}$  and  $7\frac{1}{2}$  together. Ans.  $16\frac{73}{80}$ .

## CASE V.

*When the given fractions are of several denominations.*

## RULE.

Reduce them to their proper values, or quantities, and add them according to the following examples.

## EXAMPLES.

1. Add  $\frac{2}{3}$  of a pound to  $\frac{2}{5}$  of a shilling.

Thus,  $\frac{2}{3}$  of a pound = 13s. 4d.

and  $\frac{2}{5}$  of a shilling = 0s. 4d.  $3\frac{1}{2}$ qr.

13s. 8d.  $3\frac{1}{2}$ qr. Ans.

2. Add  $\frac{7}{8}$  of a pound and  $\frac{3}{10}$  of a shilling together.

Ans. 15s.  $10\frac{4}{5}$ d.

3. Add  $\frac{1}{3}$  of a week,  $\frac{1}{4}$  of a day, and  $\frac{1}{2}$  of an hour together.

Ans. 2d.  $14\frac{1}{2}$ h.

4. Add  $\frac{3}{4}$  of a yard,  $\frac{1}{2}$  of a foot, and  $\frac{5}{8}$  of a mile together. Ans. 1160 yds. 2 ft. 7 in.

5. Add  $\frac{1}{5}$  of a dollar,  $\frac{2}{3}$  of a cent,  $\frac{3}{16}$  of a cent, and  $\frac{1}{4}$  of a mill together. Ans. 20c. 9m.

6. Add  $\frac{1}{3}$  of a pound,  $\frac{2}{7}$  of a shilling, and  $\frac{4}{5}$  of a penny together. Ans. 2s. 8  $\frac{64}{175}$  d.

## SUBTRACTION OF VULGAR FRACTIONS.

### CASE I.

*When the fractions have a common denominator.*

#### RULE.

Subtract the less numerator from the greater, and set the remainder over the common denominator, which will show the difference of the given fractions.

#### EXAMPLES.

1. Subtract  $\frac{2}{7}$  from  $\frac{4}{7}$ . Ans.  $\frac{2}{7}$ .

2. What is the difference between  $\frac{2}{3}$  and  $\frac{1}{3}$ ? Ans.  $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ .

2. Take  $\frac{5}{12}$  from  $\frac{7}{12}$ . Ans.  $\frac{2}{12} = \frac{1}{6}$ .

4. Take  $\frac{2}{3}$  from  $\frac{7}{3}$ . Ans.  $\frac{4}{3} = 1\frac{1}{3}$ .

### CASE II.

*When fractions, or mixed numbers, are to be subtracted from whole numbers.*

#### RULE.

Subtract the numerator from its denominator, and under the remainder place the denominator; then carry one to be deducted from the whole number.

#### EXAMPLES.

1. Take  $\frac{2}{3}$  from 12.

Thus. 12.

0  $\frac{2}{3}$

11  $\frac{1}{3}$  Answer.

2. Subtract  $27\frac{4}{5}$  from 32. Ans.  $4\frac{1}{5}$ .

3. From 10, take  $\frac{1}{6}$ . Ans.  $9\frac{5}{6}$ .

4. From 9, take  $5\frac{1}{2}$ . Ans.  $3\frac{1}{2}$ .

5. From 25, take  $24\frac{1}{10}$ . Ans.  $\frac{9}{10} = \frac{1}{10}$ .

## CASE VI.

*To subtract fractions having different denominators.*

## RULE.

Reduce the fractions to a common denominator, by Case VI. in Reduction, and subtract the less numerator from the greater—the difference will be the answer.

## EXAMPLES.

1. What is the difference between  $\frac{12}{20}$  and  $\frac{22}{15}$ ?

Thus  $\frac{12}{20}$  and  $\frac{22}{15}$  are equal to  $\frac{171}{300}$ ,  $\frac{288}{300}$ ,

And 88 from 171, leaves 83.

Ans.  $\frac{83}{300}$ .

2. From  $\frac{3}{7}$  take  $\frac{2}{5}$ .

Ans.  $\frac{1}{35}$ .

3. Take  $\frac{1}{2}$  from  $\frac{5}{3}$ .

Ans.  $\frac{1}{6}$ .

4. Subtract  $\frac{7}{15}$  from  $\frac{9}{20}$ .

Ans.  $\frac{1}{60}$ .

## CASE VI.

*To distinguish the largest of any two fractions.*

## RULE.

Reduce them to a common denominator, and the one that has the larger numerator is the larger fraction.

## EXAMPLE.

Which is the greater fraction,  $\frac{11}{12}$  or  $\frac{15}{16}$ ?

Thus 192 common denominator.

$12 \times 15 = 180$  numerator.

$16 \times 11 = 176$  numerator.

4 numerator

Then,  $\frac{4}{192} = \frac{1}{48}$ .

Therefore  $\frac{15}{16}$  is the greater fraction by  $\frac{1}{48}$ , Ans.

## CASE V.

*To subtract one mixed number from another, when the fraction to be subtracted is greater than that from which the subtraction is to be made.*

## RULE.

Reduce the fractions to a common denominator; subtract the numerator of the greater from the common denominator, and add to the remainder the less numerator; then set the sum of them over the common denominator, and carry one to the whole number, and subtract as in Simple Subtraction.

EXAMPLES.

1. From  $12\frac{3}{8}$  subtract  $8\frac{1}{2}$ .

Thus  $\frac{3}{8}$  reduced to a common denominator,  $=\frac{57}{112}$ ,  
and  $\frac{1}{2}$  reduced to a common denominator,  $=\frac{56}{112}$ .

Then 72 taken from 114, leaves 42; which, added to 57, the less numerator, makes 99 for the numerator in the answer. Then carrying 1 to the whole number, namely, 8, makes it 9; and taking 9 from 12 leaves 3.

Therefore the answer is  $3\frac{99}{112}$ .

2 From  $10\frac{3}{10}$ , take  $1\frac{7}{12}$ .

Ans.  $8\frac{43}{60}$ .

CASE VI.

*When fractions are of different denominations.*

RULE.

Reduce them to their proper values, or quantities, and subtract as in Compound Subtraction.

EXAMPLES.

1. From  $\frac{7}{8}$  of a pound, take  $\frac{1}{3}$  of a shilling.

Thus,  $\frac{7}{8}$  of a pound  $= 17s. 6d.$

And  $\frac{1}{3}$  of a shilling  $= 0 \quad 4$

---

17s. 2d. Answer.

---

2. From  $\frac{5}{8}$  of a ton take  $\frac{9}{10}$  of a cwt.

Ans. 14cwt. 0qr. 11lb. 3oz.  $3\frac{1}{2}$ dr.

3. From  $\frac{3}{4}$  of a pound, take  $\frac{2}{4}$  of a shilling and what will be the remainder? Ans. 14s. 3d.

4. From  $\frac{3}{4}$  of a pound, Troy Weight, take  $\frac{1}{6}$  of an ounce. Ans. 8oz. 16dwt. 16gr.

---

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

Reduce compound fractions to simple ones, and mixed numbers to equivalent fractions; then multiply all the numerators together for a numerator, and all the denominators together for a denominator which will give the product required.

## EXAMPLES.

1. Multiply
- $\frac{3}{4}$
- by
- $\frac{2}{5}$
- .

Here,  $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$  the answer.

2. Multiply
- $\frac{2}{7}$
- by
- $\frac{5}{2}$
- .

Ans.  $\frac{5}{7}$ .

3. "
- $\frac{4}{15}$
- by
- $\frac{5}{24}$
- .

Ans.  $\frac{1}{6}$ .

4. "
- $\frac{1}{2}$
- of 7 by
- $\frac{3}{8}$
- .

Ans.  $1\frac{3}{4}$ .

5. "
- $6\frac{2}{3}$
- by
- $\frac{1}{4}$
- .

Ans.  $1\frac{3}{2}$ .

6. "
- $4\frac{1}{2}$
- by
- $3\frac{2}{3}$
- .

Ans.  $14\frac{1}{2}$ .

7. "
- $4\frac{1}{2}$
- by
- $\frac{1}{4}$
- .

Ans.  $1\frac{1}{8}$ .

## DIVISION OF VULGAR FRACTIONS.

## RULE.

Reduce compound fractions to simple ones, and mixed numbers to equivalent fractions; then multiply the numerator of the dividend by the denominator of the divisor, for a new numerator, and the denominator of the dividend by the numerator of the divisor, for the denominator; the fractions thus formed will be the answer.

## EXAMPLES.

1. Divide
- $\frac{4}{7}$
- by
- $\frac{3}{5}$
- .

Thus, 4 numerator of the dividend.

3  $\times$  denominator of the divisor.

12 numerator.

Then 7 denominator of the dividend.

2  $\times$  numerator of the divisor.

14 denominator

Therefore,  $\frac{12}{14} = \frac{6}{7}$  is the answer.

2. Divide
- $\frac{2}{3}$
- by
- $\frac{3}{4}$
- .

Thus,  $2 \times 4 = 8$  $3 \times 3 = 9$  Answer.

3. Divide
- $1\frac{1}{2}$
- by
- $\frac{4}{5}$
- .

Ans.  $1\frac{5}{8}$ .

4. "
- $\frac{7}{8}$
- by
- $\frac{3}{4}$
- .

Ans.  $1\frac{1}{8}$ .

5. "
- $\frac{5}{6}$
- by
- $1\frac{1}{2}$
- .

Ans.  $1\frac{2}{3}$ .

6. "
- $\frac{1}{2}$
- by
- $\frac{3}{4}$
- .

Ans.  $\frac{2}{3}$ .

7. "
- $\frac{3}{4}$
- by
- $\frac{3}{4}$
- .

Ans.  $1\frac{1}{4}$ .

8. "
- $\frac{1}{6}$
- by 3.

Ans.  $\frac{1}{2}$ .



9. Divide  $\frac{2}{5}$  by 2. Ans.  $\frac{3}{10}$ .  
 10. "  $7\frac{1}{3}$  by  $9\frac{5}{9}$ . Ans.  $\frac{23}{43}$ .  
 11. "  $\frac{2}{3}$  of  $\frac{1}{3}$  by  $\frac{5}{7}$  of  $7\frac{2}{5}$ . Ans.  $1\frac{7}{11}$ .  
 12. What part of  $33\frac{1}{21}$ , is  $28\frac{11}{12}$ ? Ans.  $\frac{7}{8}$ .

**Questions.**

1. What are Vulgar Fractions?
2. How are they represented in figures?
3. What is the upper figure called?
4. What is the lower figure called?
5. What does the denominator show?
6. What does the numerator show?
7. What are the two numbers of a fraction sometimes called?

---

## DECIMAL FRACTIONS.

Decimal Fractions are parts of whole numbers, and are separated from them by a point, thus, 8.5; which is read, eight and five tenths, or  $8\frac{5}{10}$ . All the figures on the left of the point are whole numbers; those on the right are fractions. An unit is supposed to be divided into ten equal parts, and the figure at the right of the point expresses the number of those parts. Decimals decrease in a tenfold proportion, as they depart from the separating point. Thus, .5 is 5 tenths, or one half; .57 is 57 hundredths; .05 is 5 hundredths; and .005 is 5 thousandths. Cyphers placed at the right hand of decimals do not alter their value; thus, .5 or  $\frac{5}{10}$ ; .50 or  $\frac{50}{100}$ ; .500 or  $\frac{500}{1000}$ , are all of the same value, and equal to  $\frac{1}{2}$ . The first place of decimals is called tenths; the second, hundredths, &c.

**DECIMAL NUMERATION TABLE.**

7	6	5	4	3	2	1	6	5	4	3	2	1
Millions.	Hundred thous.	Ten thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thous.	Millionths.

## ADDITION OF DECIMALS.

## RULE.

Place the figures according to their values—units under units, tenths under tenths, &c. and add as in Simple Addition of whole numbers; observing to place the point in the sum under those in the given numbers.

## EXAMPLES.

1. Add together the following sums, viz: 252.25, 343.5, 17.85, 1244.75 and .425.

Thus, 252.25      *Note.*—The answer to this sum  
          343.5      is read thus: One thousand eight  
          17.85      hundred and fifty eight, and seven  
          1244.75      hundred and seventy-five thou-  
          .425      sandths.

1858.775 Answer.

## II.

87654.321  
 23456.78  
 98765.4

209876.501

## III.

987654.3  
 212345.67  
 898765.432

2098765.402

4. Add 420.4, 38.05, 54.9, 27.003 and 29.384.

Ans. 569.737.

5. Add 376.25, 86.125, 6.5, 41.02 and 358.865.

Ans. 868.760.

6. Add .64, .840, .4, .04, .742, .86, .99 and .450.

Ans. 4.962.

7. William expended for a gig \$255  $\frac{2}{10}$ , for a wagon \$37  $\frac{25}{100}$ , for a bridle  $\frac{75}{100}$  and for a saddle \$11  $\frac{255}{1000}$ .

What did they amount to?

Ans. \$304.455.

*Note.*—Dimes, cents and mills are decimals of a dollar. A dime is one tenth, a cent is one hundredth, a mill is one thousandth; which shows that the addition of Federal Money is the addition of decimals. Thus 5 tenths of a dollar is the same as 50 hundredths, or 50 cents; and 25 hundredths of a dollar is equal to 25 cents, &c. It may be likewise added, that .5, or .50, or .500 being equal to one half, .25 equal to one quarter, and .75 equal to three quarters or three fourths, so .7, or .35, or any intermediate fractions, have a proportionate value.

8. James bought 2.5 cwt. of sugar, 23.265 cwt. of hay and 4.2657 of rice. How much did he buy in all?

Ans. 30.0307cwt.

9. James is  $14\frac{5}{10}$  years old, John  $15\frac{25}{100}$  and Thomas  $16\frac{75}{100}$ . What is the sum of all their ages?

Ans. 46.5 years.

10. What is the sum of one and five tenths; forty-five and three hundred and forty nine thousandths; and sixteen hundredths?

Ans. 47.009.

## SUBTRACTION OF DECIMALS.

## RULE.

Write the larger number first, and the smaller one under it; then subtract as in Simple Subtraction; observing, that the dividing point in the answer, or remainder, must be placed under those in the sum.

## EXAMPLES.

I.	II.	III.
From 91.73	2.73	1.5
take 2.138	1.9185	.987654321
<hr/>	<hr/>	<hr/>
89.592	0.8115	0.512345679
<hr/>	<hr/>	<hr/>

4. Bought a hogshead of molasses, containing 60.72 gallons and sold 40.721 gallons. How much was left in the hogshead?

Ans. 19.999 galls.

5. A merchant, owing \$270.42, paid \$191.626; how much did he then owe?

Ans. 78.794.

6. I bought 20.25 yards of cloth and sold 5.75 yards. How much had I left?

Ans. 14.50 yds.

7. From .650 of a barrel take .6 of a barrel.

Ans. .050 barrel.

8. From  $\frac{226}{1000}$  of a bushel take  $\frac{1}{10}$  of a bushel.

Ans. .126 bushel.

9. A farmer was 42.075 years old when he came to the Western country, and is now 64.67 years of age. How long since he emigrated?

Ans. 22.595 years.

10. Take one hundredth from on tenth. Ans. .09.

**MULTIPLICATION OF DECIMALS.****RULE.**

Place the multiplier under the multiplicand, and multiply as in Simple Multiplication; then point off as many places for decimals as there are decimals in the multiplicand and multiplier. If there be not so many figures in the product as there are decimals in both factors, the deficiency must be supplied by prefixing cyphers.

**EXAMPLES.**

I.	II.	III.
Multiply 24.85 by 6.25	456.75 8.75	.63478 .8994
<hr/>	<hr/>	<hr/>
12425	228375	253912
4970	319725	571302
14910	365400	571302
<hr/>	<hr/>	<hr/>
155.3125	3996.5625	507824
<hr/>	<hr/>	<hr/>
		570921132

4. Multiply .63478 by .8204.    Ans. .520772512.

5. Multiply .385746 by .00464.    Ans. .00178986144.

6. What will 5.66 bushels of wheat cost at \$1.08 a bushel?    Ans. \$6.1128, or \$6.11c.  $2\frac{1}{10}$  m.

7. What will 8.6 pounds of flour come to at \$.04 a pound?    Ans. \$.34cts. 4m.

8. At \$ .25c. a bushel, what will 12.67 bushels of apples cost?    Ans. 3.1675.

9. If I travel 30.75 miles a day, how far shall I travel in 8.325 days?    Ans. 255.99375 miles.

**DIVISION OF DECIMALS.****RULE.**

Divide as in Simple Division, and point off as many figures from the right hand of the quotient, for decimals, as the decimal figures in the dividend exceed in number those in the divisor. When there are not so many figures in the quotient as this rule requires, the

deficiency must be supplied by prefixing cyphers to the left of the quotient. When there are more decimal figures in the divisor than in the dividend, place as many cyphers to the right of the dividend as will make them equal.—When the number of decimals in the divisor, and the number in the dividend are equal, the quotient will always be in whole numbers, unless there should be a remainder after the dividend is all brought down. When there is a remainder, cyphers must be annexed to it and the division continued and the quotient thence arising will be decimals.

## EXAMPLES.

I.	II.	III.
$\begin{array}{r} .5)75(1.5 \\ \underline{5} \\ 25 \\ \underline{25} \\ - \end{array}$	$\begin{array}{r} 324.8)9876.5(30.4079 \\ \underline{9744} \\ 13250 \text{ cypher} \\ \underline{12992} \text{ annexed.} \\ 25800 \\ \underline{22736} \\ 30640 \\ \underline{29232} \\ 1408+ \end{array}$	$\begin{array}{r} .48)65.88(137 \\ \underline{48} \\ 178 \\ \underline{144} \\ 348 \\ \underline{336} \\ 12 \text{ rem.} \end{array}$
$\begin{array}{r} \text{IV.} \\ 6.4)128.64(20.1 \\ \underline{128} \\ 64 \\ \underline{64} \\ - \end{array}$		

5. Divide 234.70525 by 64.25.

Ans. 3.653.

6. Divide 14 by .7854.

Ans. 17.825.

7. Divide 2175.68 by 100.

Ans. 21.7568.

8. If you divide 116.5 barrels of flour equally among 5 men, how many barrels will each have?

Ans. 23.3 barrels.

9. At \$ .25 a bushel, how many bushels of corn may be bought for \$300.50.

Ans. 1202 bushels.

10. At \$ .12½ or \$ .125 a yard, how many yards of cloth may be bought for \$16?

Ans. 128 yds.

11. Bought 128 yards of tape, for \$ .64; how much was that a yard?

Ans. \$ .005, or 5 mills.

## REDUCTION OF DECIMALS.

## CASE I.

*To reduce a vulgar fraction to a decimal.*

## RULE.

Place cyphers to the right of the numerator until you can divide it by the denominator; and divide till nothing remains; or, if it be a number that will not divide without a remainder, then divide until you get three or more figures for the quotient. The quotient will be the vulgar fraction expressed in decimals.

## EXAMPLES.

1. Reduce  $\frac{1}{2}$  to a decimal.

$$\begin{array}{r} \text{Thus, } 2 \overline{)1.0(.5} \\ \underline{10} \end{array}$$

2. Reduce  $\frac{1}{4}$  and  $\frac{3}{4}$  to decimals.

$$\begin{array}{r} 4 \overline{)1.00(.25} \\ \underline{8} \end{array}$$

$$\begin{array}{r} 4 \overline{)3.00(.75} \\ \underline{28} \end{array}$$

20

20

20

20

3. Reduce  $\frac{3}{8}$  to a decimal.

Ans. .375.

4. Change  $\frac{1}{8}$ ,  $\frac{3}{8}$ ,  $\frac{5}{8}$ , and  $\frac{1}{2}$  to decimals.

Ans. .125, .5, .75, .04.

5. What decimal is equal to  $\frac{1}{20}$ ? Ans. .05. What is equal to  $\frac{1}{5}$ ? Ans. .2. What is equal to  $\frac{1}{3}$ ? Ans. .3333+.

6. Change  $64\frac{3}{4}$  bushels to its equivalent value.

Ans. 64.25 bushels.

## CASE II.

*To reduce any sum, or quantity, to the decimal of any given denomination.*

## RULE.

Reduce the quantity to the lowest denomination, and reduce the proposed integer to the same denomination; then divide the quantity by the amount of the integer, and the quotient will be the answer.

## EXAMPLES.

1. Reduce 3s. 9d. to the fraction of a pound.  
One pound reduced to pence makes 240; and 3s. 9d reduced to pence makes 45.

Then,  $240 \div 45 = 0.0000$  (.1875 Answer.

240

2100

1920

1800

1680

1200

1200

The same sum may be done by writing the given numbers from the least to the greatest in a perpendicular column, and dividing each of them by such number as will reduce it to the next denomination, annexing the quotient to the succeeding number.

Thus,  $12 \over 9.00$

$2 \over 0 \mid 3.750 \mid 0$

.1875 Answer.

2. Reduce 7 drams to the decimal of a pound, Avoirdupois Weight. Ans. .02734375.

3. Reduce 14 minutes to the decimal of a day. Ans. .009722.

4. Reduce 21 pints to the decimal of a peck. Ans. 1.3125.

5. Reduce 15s. 6d. to the decimal of a pound. Ans. .775.

6. Reduce 56 gallons 3 quarts 1 pint to the decimal of a hogshead. Ans. .9027.

7. Reduce 12 dwts. 16grs. to the decimal of a pound Troy Weight. Ans. .0527.

8. Reduce 4 mills to the decimal of a dollar. Ans. .004.

*Note.*—In doing sums in this rule, it will be necessary to keep in mind the tables of the different weights, measures, money, &c.

## CASE III.

*To find the value of any decimal fraction.*

## RULE.

Multiply the decimal by the number of parts in the next lower denomination; point off as many figures for decimals as is required by the rule in multiplication of decimals; then multiply the decimal by the number of parts in the next lower denomination, and so on, to the last. The figures on the left of the points will show the value of the decimal in the different denominations.

## EXAMPLES.

1. What is the value of .775 of a pound?

£. 775

20

—  
s. 15.500

12

—  
d. 6.000 Answer.

2. What is the value of .625 of a cwt.?

4

—  
2.500

28

—  
4000

1000

—  
14.000 Ans. 2 qr. 14lb.

3. What is the value of .625 of a shilling? Ans. 7½d.

4. What is the value of .4694 of a pound, Troy Weight? Ans. 5oz. 12dwts. 15.744grs.

5. What is the value of .6875 of a yard?

Ans. 2qrs. 3na.

6. What is the value of .3375 of an acre?

Ans. 1R. 14P.

7. What is the value of .0008 of an Eagle?

Ans. 8m.



*Questions.*

1. What are decimal Fractions?
2. How are they separated from whole numbers?
3. In what manner do they decrease as they depart from the separating point?
4. In the table of numeration, what is the first place called?
5. What money, or currency, is reckoned after the manner of Decimal Fractions?

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## DUODECIMALS.

Duodecimals are fractions of a foot or of an inch, or parts of an inch, and have 12 for their denominator.—They are useful in measuring planes, or surfaces, and solids. In adding, subtracting, and multiplying by Duodecimals, it is necessary to carry one for twelve.

The denominations are feet, inches, seconds, thirds and fourths.

12 fourths''''	make	1 third	'''.
12 thirds -	-	1 second	''.
12 seconds -	-	1 inch	I.
12 inches -	-	1 foot	Ft.

---

## MULTIPLICATION OF DUODECIMALS.

**RULE.**

Set down the different denominations, one under the other, so that feet stand under feet, inches under inches, seconds under seconds, &c. Multiply each denomination in the sum, by the feet in the multiplier, and set the result of each under its corresponding term, observing to carry one for every 12 from one denomination to another. Then multiply the sum by the inches in the multiplier, and set the result of each term one place removed to the right of those in the sum; and in like manner, multiply the sum or multiplicand by seconds, thirds, &c. if there be any in the multiplier.

Or, instead of multiplying by inches, &c. take such parts in the multiplicand, as these are of a foot.

Add the amount of the multiplications together, and their sum will be the answer.

## EXAMPLES.

I.

	<i>Ft.</i>	<i>I.</i>
Multiply	4	7
by	6	4
<hr/>		
	27	6
	1	6 4
<hr/>		
	29	0 4

II.

	<i>Ft.</i>	<i>I.</i>
	14	9
	4	6
<hr/>		
	59	0
	7	4 6
<hr/>		
	66	4 6

*Note 1.*—Feet multiplied by feet, give feet. Feet multiplied by inches give inches. Feet multiplied by seconds, give seconds. Inches multiplied by inches, give seconds. Inches multiplied by seconds, give thirds. Seconds multiplied by seconds, give fourths.

III.

	<i>Ft.</i>	<i>I.</i>	"	'''
Multiply	8	4	2	10.
by	4	2		
<hr/>				
	33	4	11	4
	1	4	8	5 8
<hr/>				
	34	9	7	9 8'''

IV.

	<i>Ft.</i>	<i>I.</i>
	11	10.
	10	9
<hr/>		
	118	4
	8	10 6
<hr/>		
	127	2 6''

*Note 2.*—In doing the third sum, I begin with 4, which stands under the 8, and multiply the sum, beginning with the right hand figure which is 10; saying 4 times 10 are 40. In 40, I find there are 3 times 12 and 4 over. Setting down 4, I multiply the next figure, adding three to it, which makes 11, and thus multiply the whole sum. Then taking the 2 for the multiplier, I say 2 times ten are 20. In 20 I find 12 is contained once, and 8 over. Setting down 8 one place farther to the right, I say 2 times 2 are 4, and one to carry makes 5; and after this manner multiply all the figures in the sum. Then adding the two rows of figures together, I obtain the answer

Method of doing the same sum by taking the fractional parts.

	<i>Ft.</i>	<i>I.</i>	<i>"</i>	<i>'''</i>	
2 inches = $\frac{1}{6}$	8	4	2	10	
	4	2			
	33	4	11	4	
	1	4	8	5	8
	34	9	7	9	8''' Answer.

In this last example, I multiply the sum by 4, as in the former case. Then, as 2 inches make  $\frac{1}{6}$  of a foot, I divide the sum by 6, which I had multiplied by 4, dividing it after the manner of Compound Division, multiplying each remainder by 12, and adding it to the next lower denomination; and setting the result under the amount of the multiplication. Then I add the two sums as before.

5. What are the solid contents of a cubick block that is 4 feet 4 inches in length, 3 feet 8 inches in breadth, and 2 feet 8 inches in thickness?

Ans. 42*Ft.* 4*I.* 5" 4'''.

6. What is the product of 12 feet 9 inches, multiplied by 6 feet 4 inches.

Ans. 80*Ft.* 9*I.*

7. What is the product of 3 feet 2 inches 3" multiplied by 3 feet 2 inches 3"?

Ans. 10*Ft.* 11*I.* 11" 0''' 9'''.

8. What is the price of a marble slab whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at one dollar per foot?

Ans. \$10.23.

9. How many square feet in a board 17*Ft.* 7*I.* long and 1*Ft.* 5*I.* wide?

Ans. 24*Ft.* 19*I.* 11"

10. How many solid feet in a load of wood 6*Ft.* 7*I.* long and 3*Ft.* 5*I.* high and 3*Ft.* 8*I.* wide?

Ans. 82*Ft.* 5*I.* 8" 4'''.

11. What will be the expense of plastering the walls of a room 8*Ft.* 6*I.* high and each of the 4 sides 16*Ft.* 3*I.* long, at 50 cents per square yard?

Ans. \$30.690+

*Questions.*

1. What are Duodecimals?
  2. In what are they useful?
  3. In adding, subtracting, and multiplying Duodecimals, what do you observe in carrying from one denomination to another?
  4. What are the denominations used in Duodecimals?
  5. Repeat the rule for Multiplication of Duodecimals?
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**SINGLE RULE OF THREE.**

The Rule of Three, which is sometimes called the Rule of Proportion, teaches how to find a fourth proportional to three numbers given. As it has three terms given to find a fourth, it is generally called the Rule of Three.

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*Questions to prepare the learner for this rule.*

1. If 2 apples cost 3 cents, how much will 4 apples cost at the same rate?
2. If you give 2 cents for 4 nuts, how many cents must you give for 8 nuts?
3. If a pound of butter cost 8 cents, how much will 4 pounds cost?
4. A boy has 20 melons to sell, and asks 10 cents for two, how much will they all come to at the same rate?
5. If 6 men can reap a field of wheat in 4 days, how long will it take 12 men to reap the same field?
6. If 4 yards of cloth cost 1 dollar, how much will 2 yards cost?
7. How much will a gallon of milk come to, at four cents a quart?
8. How much will a bushel of peaches come to at 25 cents a peck?
9. If 3 cents will buy 2 apples, how many apples will 9 cents buy?
10. If a boy can run 2 miles in one hour, how far can he run in 4 hours?

## RULE.

Set the term in the first place, which is of the same kind with that in which the answer is required. Then determine whether the answer ought to be greater or less than the third term. If the answer ought to be greater than the third term, set the greater of the other two numbers on the left for a second or middle term; and the less number on the left of the second term, for a first term. If the answer ought to be less than the third term, the less of the two other numbers must be the middle term, and the greater must be the first term.

After thus stating the sum, proceed to do it in the following manner, viz: Reduce the third term to the lowest denomination mentioned in it. Reduce, likewise, the first and second terms to the lowest denomination that either of them has. Then multiply the second and third terms together, and divide their product by the first term. The quotient thus obtained will be the answer.

It will not be necessary to distinguish between direct and inverse proportion, because the foregoing rule is calculated for both.

## PROOF.

By reversing the statement.

## EXAMPLES.

1. If 3 pounds of sugar cost 25 cents, what will 18 pounds cost at the same rate?

	<i>lbs.</i>	<i>lbs.</i>	<i>cts.</i>
Thus,	3	: 18	: : 25
			18

---

200  
25

---

3)450

---

\$1.50 Answer.

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2. If 7 pounds of coffee cost  $87\frac{1}{2}$  cents, what must I pay for 244 pounds?

$$\begin{array}{rcl} \text{lbs.} & \text{lbs.} & \text{cts.} \\ \text{Thus, } 7 & : 244 & : : 87\frac{1}{2} \end{array}$$

$$\begin{array}{r} 1708 \\ 1952 \\ 122 \\ \hline 7)21350 \end{array}$$

$\$30.50$  Answer.

3. If 450 barrels of flour cost \$1350, what will 8 barrels cost?

$$\begin{array}{rcl} \text{bbls.} & \text{bbls.} & \$ \\ \text{Thus—As } 450 & : 8 & : : 1350 : 24, \text{ Answer.}^* \end{array}$$

4. If 15 yards of cloth cost £8, what number of yards may be bought for £125?

$$\begin{array}{rcl} \text{£} & \text{£} & \text{yds.} \\ \text{As } 6 & : 125 & : : 15 : 312\frac{1}{2} \text{ Answer.} \end{array}$$

5. If twelve men can do a piece of work in 20 days, in what time will 18 men do it?

$$\begin{array}{rcl} \text{m.} & \text{m.} & \text{d.} \\ \text{As } 18 & : 12 & : : 20 : 13\frac{1}{3} \text{ Answer.} \end{array}$$

6. What will be the cost of 17 tons of lead, at 223 dollars 66 cents per ton?

$$\begin{array}{rcl} \text{T.} & \text{T.} & \text{D. cts.} \\ \text{As } 1 & : 17 & : : 223.66 : 3802.22 \text{ Ans.} \end{array}$$

7. What will 72 yards of cloth cost at the rate of 9 yards for £5 12s.

$$\begin{array}{rcl} \text{yds.} & \text{yds.} & \text{£ s.} \\ \text{As } 9 & : 72 & : : 5 \text{ } 12 : 44 \text{ } 16 \text{ Ans.} \end{array}$$

8. If 750 men require 22500 rations of bread for a month, what will a garrison of 1200 require?

Ans. 36000.

\*The sum in the third example is read thus:—As 450 is to 8, so is 1350 to the answer. This is the manner of reading all sums when stated in the Rule of Three.

9. What must be the length of a board that is 9 inches in width, to make a surface of 144 inches, or a square foot?

Ans. 16 inches.

10. How many yards of matting 2 feet 6 inches broad, will cover a floor that is 27 feet long, and 20 broad?

Ans. 72 yards.

11. If a person's annual income be 520 dollars, what is that per week?

Ans. 10 dollars.

12. If a pasture be a sufficient for 3000 horses for 18 days, how long will it be sufficient for 2000?

*H. H. D. D.*

As 2000 : 3000 : : 18 : 27 Ans.

13. What must be the length of a piece of land 13½ rods in breadth, to contain one acre?

Ans. 11 rods, 4yds, 2ft. 0½ in.

14. If 8 men can build a tower in 12 days, in what time can 12 build it?

*M. M. D. D.*

As 12 : 8 : : 12 : 8 Answer.

15. If a piece of land be 5 rods in width, what must be its breadth to make an acre?

*R. R. R. R.*

As 5 : 160 : : 1 : 32 Answer.

16. How much carpeting that is 1½ yards in breadth, will cover a floor 7½ yards in length, and five yards in breadth?

*By Decimal Fractions.*

*yds. yds. yds.*

As 1.5 : 5 : : 7.5 : 25 Answer.

17. What will one quart of wine cost at the rate of 12 dollars for 16 gallons?

*gals. qts. qt. D. cts.*

As 16 or 64 : 1 : : 12.00 : 18¾ Ans.

18. If 10 pieces of cloth, each piece containing 42 yards, cost 531 dollars 30 cents, what does it cost per yard?

Ans. \$1.26½

19. If a hogshead of brandy cost 78 dollars 75 cents, what must be given for 5 gallons at the same rate?

Ans. \$6.25

20. If a staff 4 feet in length, cast a shade on level ground, 8 feet in length, what is the height of a tower, whose shade, at the same time, measures 200 feet?

*ft. ft. ft. ft.*  
As 8 : 200 : : 4 : 100 Answer.

21. I lent my friend 350 dollars for 5 months, he promising to do me the same favour; but when requested, he could spare only 125 dollars. How long ought I to keep it to balance the favour?

*D. D. M. M.*  
As 125 : 350 : : 5 : 14 Answer.

22. If 7 oxen be worth 10 cows, how many cows will 21 oxen be worth?

*Ox. Ox. C. C.*  
As 7 : 21 : : 10 : 30 Answer.

23. If board for 1 year, or 52 weeks, amount to \$182, what will 39 weeks come to? Ans. \$133.50.

24. If 30 bushels of rye may be bought for 120 bushels of potatoes, how many bushels of rye may be bought for 600 bushels of potatoes? Ans. 150 bushels rye.

25. A gentleman bought a bag of coffee for the use of his family, weighing 127 lbs., for which he gave \$15.25 cts.; what was it a pound? Ans. 12 + cts.

26. If a gentleman spends 4 dols. 62½ cts. every day, how much will that amount to in a year?

Ans. 1688.12½.

27. A lady bought for the use of her family, a piece of cloth, containing 16 yds. 3 qrs. 2 na., at \$1.25 cts. a yard; what was the amount of the whole?

Ans. \$21.09 +

28. A, failing in trade, owes the following sums, viz. to B \$1600.60, to C \$500, to D \$750.20, to E \$1000, to F \$230; and his property, which is worth no more than \$1020.20, he gives up to his creditors; how much does he pay on the dollar? and what is the amount of loss sustained by all?

Ans. 25 cts. on \$1; and the amount of loss is \$3060.60.

29. A farmer made from an orchard of apples, 146 barrels of cider, which he afterwards sold at \$3.12½ cts. a barrel; what was the amount of the whole?

Ans. \$456.25.



30. A flour merchant sold 1574 bbls. of flour, at \$5.12 $\frac{1}{2}$  cents a barrel, what was the amount? Ans. \$8066.75.

31. A merchant failing, was able to pay his creditors only 62 $\frac{1}{2}$  cents in a dollar; how much will a person receive, whose claim is 746.25 cents?

Ans. \$466.40,6,25.

32. A lady purchased a set of silver, weighing in the whole 5 lb. 6 oz. 5 dwt. at 1.50 cents an ounce; what was the cost of the whole?

Ans. \$93.37,5.

33. A lady, intending to make a bed quilt, containing 8 $\frac{1}{2}$  square yards, desired her daughter to inform her how much muslin 1 $\frac{1}{4}$  yard wide would be required to line the same; how many did it take? Ans. 6.8 yds. or 6 $\frac{1}{2}$  yds.

34. A certain field will afford pasture for 10 oxen, for 60 days; how long will the same pasture suffice for 24 oxen?

Ans. 25 days.

35. A pipe will drain off a cistern of water in 12 hours; how many pipes of the same size will empty it in 30 minutes?

Ans. 24 pipes.

36. Two persons travel on the same road, and in the same direction. One sets out 5 days before the other, and travels 20 miles a day; the other travels 25 miles a day; how long before the latter will overtake the former?

Ans. 20 days.

37. If 48 men can build a fortification in 24 days, how many men can do the same in 192 days? Ans. 6 men.

38. A certain piece of work was done by 120 men in 8 months, how many men will it take to do another piece of work of the same magnitude in 2 months?

Ans. 480.

39. A merchant failing in trade, owes 29475 dollars: he delivers up his property, which is worth 21894 dollars 3 cents; how much does this sum pay on the dollar towards what he owes?

Ans, 74cts. 2m.+

40. If property rated at 28 dollars, pay a tax of 21 dollars, how much is that on the dollar?

Ans. 75 cents.

41. If  $\frac{1}{4}$  of a yard cost 62 $\frac{1}{2}$  cents, what will  $\frac{7}{8}$  of a yard cost?

Ans. \$2.187.+

42. If a tax of 30,000 dollars be laid on a town in which the ratable property is estimated at 9,000,000 dollars, what will be the tax of one of the citizens whose ratable estate is reckoned at 750 dollars?

*D.*                      *D.*                      *D.*                      *D. cts.*

As 9,000,000 : 30,000 . : 750 : 2 : 50 Ans.\*

43. How far are the inhabitants on the equator carried in a minute, allowing the earth to make one revolution in 24 hours; and allowing a degree to contain  $69\frac{1}{2}$  miles?

The earth being divided into 360 degrees, allowing  $69\frac{1}{2}$  miles to a degree, makes the distance round it to be 25020 miles;—the number of minutes in 24 hours is 1440.

Ans. 17 miles, 3 fur.

44. There is a cistern having 4 spouts; the first will empty in 15 minutes, the second in thirty minutes, the third in 45 minutes, and the fourth in 60 minutes: in what time would the cistern be emptied, if they were all running together?

As 15 : 1 : : 90 : 6

30 : 1 : : 90 : 3

45 : 1 : : 90 : 2

60 : 1 : : 90 :  $1\frac{1}{2}$

---

$12\frac{1}{2}$

---

*cisterns. cist.*                      *min. min. sec.*

Then, decimally, as 12.5 : 1 : : 90 : 7.12 Ans.

\*In making taxes in a due proportion, according to the value of each man's ratable estate, proceed in the following manner. Make the amount of ratable property the first term; make the sum to be raised the second term; and one dollar the third term; and the number arising from this operation will be the amount to be raised on the dollar. From this, make a tax table from one dollar to 30, or any amount necessary. In the same manner find what is to be paid on a cent of ratable estate; and from this, make a table from 1 to 99 cents; then, from these tables, take each man's tax. Thus, if the tax were 75 cents on the dollar, and you would know what a portion of property pays, that is rated at \$28.80, the tables will show the amount to be \$21, for the dollars, and 60 cts., for the cents. In estimating property for making taxes, it is customary to rate it much lower than its real value.

45. If a ship's company of 15 persons have a quantity of bread, sufficient to afford to each one 8 ounces per day, during a voyage at sea, what ought to be their allowance, under the same circumstances, if 5 persons be added to their number?      Ans. 6 ounces.

*Note.*—As the Rule of Three in Vulgar and Decimal Fractions require the same statements as in whole numbers, and is performed by multiplication and division after the same manner of other sums in the Rule of Three, it is deemed unnecessary to give any examples. When the pupil understands *Fractions and the Rule of Three*, he will find no difficulty with the Rule of Three in Fractions.

- Q. 1. What is the Rule of Three sometimes called?  
 2. What does it teach?  
 3. Which of the terms must be set in the third place?  
 4. How do you ascertain which ought to be the first term, and which is the second?  
 5. If the third term consist of different denominations, what do you do with them?  
 6. What do you do if the first and second terms are of different denominations?  
 7. After stating the sum, and reducing, when necessary, the terms to similar denominations, how do you proceed to do the sum?  
 8. How are sums in the Single Rule of Three proved?

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## DOUBLE RULE OF THREE.

The Double Rule of Three is that in which five or more terms are given to find another term sought.

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### RULE.

Set the term which is of the same denomination as the term sought, in the third place; then consider each pair of similar terms separately, and this third one, as making the terms of a statement in the Single Rule of Three, setting the similar terms in the first or second places, according to the rule of the Single Rule of Three. After stating the question in this manner, and reducing,

if necessary, the similar terms to similar denominations, then multiply the terms in the second and third places together for a dividend, and the terms in the first place together for a divisor—the quotient, after dividing, will be the term sought.

Sums in this rule may also be done by two or more statements in the Single Rule of Three.

## PROOF.

By inverting the statement, or, more easily, by two statements in the Single Rule of Three.

## EXAMPLES.

1. If 8 men, in 16 days, can earn 96 dollars, how much can 12 men earn in 26 days?

$$\begin{array}{rcll} \text{men} & 8 & : & 12 & : & : & \} \\ \text{days} & 16 & : & 26 & : & : & \} \end{array} \quad \$96$$

$$\begin{array}{r} 128 \\ 312 \\ \hline 96 \times \end{array}$$

$$\begin{array}{r} 1872 \\ 2808 \\ \hline \end{array}$$

$$\begin{array}{r} \text{---} \text{---} D. \\ 128 \overline{) 29952} (234 \text{ Answer.} \\ 256 \end{array}$$

$$\begin{array}{r} 435 \\ 384 \\ \hline \end{array}$$

$$\begin{array}{r} 512 \\ 512 \\ \hline \end{array}$$

2. If \$100 gain \$6 in 12 months, what will \$400 gain in 9 months?

$$\begin{array}{rcll} \text{As} & 100 & : & 6 & : & : & \} \\ \text{months} & 12 & : & 9 & : & : & \} \end{array} \quad \$400$$

$$\begin{array}{r} 1200 \\ 54 \\ \hline 400 \times \end{array}$$

$$12 \overline{) 00} 216 \overline{) 00}$$

\$18 Answer.

3. If 16 men can dig a trench 54 yards in length in 6 days, how many men will be necessary to complete one 135 yards in length, in 8 days?

*By two statements in the Single Rule of Three.*

yds.	yds.	men.	men.
As 54	: 135	: : 16	: 40.

days.	ds.	men.	men.
-------	-----	------	------

Then, as 8 : 6 : : 40 : 30 Answer.

4. If \$100 in one year gain \$5 interest, what will be the interest of \$750 for 7 years? Ans. \$262.50.

5. If 9 persons expend \$120 in 8 months, how much will 24 persons spend in 16 months at the same rate?

Ans. \$640.

6. If 54 dollars be the wages of 8 men for 14 days, what must be the wages of 28 men for 20 days at the same rate?

Ans. \$270.

7. If a horse travel 130 miles in 3 days, when the days are 12 hours in length, in how many days of 10 hours each can he travel 360 miles? Ans.  $9\frac{2}{3}$  days.

8. If 60 bushels of corn can serve 7 horses 28 days, how many days will 47 bushels serve 6 horses?

Ans.  $25\frac{2}{3}$  days.

9. If a barrel of beer serve 7 persons for 12 days, how many barrels will be sufficient for 14 persons for a year, or 365 days?

Ans.  $60\frac{1}{2}$  barrels.

10. If 8 men spend 32 dollars in 13 weeks, what will 24 men spend in 52 weeks?

Ans. \$384.

Q. 1. How many terms are generally given in the Double Rule of Three?

2. Which of the terms must be set in the third place?

3. How do you ascertain which of the other terms should be placed in the first, and which in the second place?

4. Which of the terms do you multiply together for a dividend?

5. How do you form a divisor?

6. How do you proceed when the terms consist of different denominations?

7. How is a sum in the Double Rule of Three proved?

*Promiscuous questions in Simple and Compound Proportion.*

1. What can you buy 15 tons of hay for, if 3 tons cost \$36? Ans. \$180.

2. William's income is \$1500 a year, and his daily expenses are \$2.50; how much will he have saved at the year's end? Ans. \$587.50.

3. If 7 men can reap 84 acres of wheat in 12 days; how many can reap 100 acres in 5 days? Ans. 20 men.

4. If a horse will trot in a gig 8 miles in an hour, how far will he trot at the same rate, in  $3\frac{1}{2}$  hours?

Ans. 28 miles.

5. A merchant bought 5 pieces of muslin, each containing 26 yards, at 11 cents a yard; what did they amount to? Ans. \$14.30.

6. If a family of 8 persons, in 24 months, spend \$480, how much would 16 persons spend in 8 months?

Ans. \$320.

7. A merchant, owning  $\frac{2}{3}$  of a vessel, sells  $\frac{1}{3}$  of his share for \$500; what was the whole vessel worth?

$\frac{2}{3}$  of  $\frac{2}{3} = \frac{4}{9} = \frac{2}{3}$ ; then, as  $\frac{2}{3}$  of the vessel is \$500,  $\frac{1}{3}$  is \$250, and  $\frac{2}{3}$ , or the whole vessel, is  $5 \times 250 = \$1250$ .

Or thus;  $\frac{2}{3}$  of  $\frac{2}{3} : 1 :: 500 : \$1250$ .

Ans., as before.

8. If  $1\frac{1}{2}$  lb. indigo cost \$3.84, what will 49,2 lbs. cost at the same rate? Ans. \$125,952.

9. If 112 acres of meadow be mowed over by 16 men, in 7 days; how many acres can 24 men mow over, in 19 days? Ans. 456 acres.

10. If 8 cwt. of iron can be carried 128 miles for \$12.80, what will be the expense of carrying 4 cwt. 32 miles? Ans. \$1.60.

11. A merchant bought a bale of cloth, containing 375 yds. at \$3.12 $\frac{1}{2}$  a yard; what did the whole amount to? Ans. \$1171.87,5.

12. A mother allows her daughter, at a boarding school, 3 cents a day for spending money; how much will that amount to in a year? Ans. \$10.95.

13. Suppose the wages of 6 persons for 21 weeks be 288 dollars, what must 14 persons receive for 46 weeks?

Ans. \$1472.

14. If 1 hundred weight of sugar cost 13 dollars 50 cents, what must be paid for 17cwt. 3qrs. 14lb.?

Ans. \$241.31 $\frac{1}{4}$  cents.

15. How many yards of paper, 2 $\frac{1}{2}$  feet wide, will be required to cover a wall, which is 12 feet long, and 9 feet high?

Ans. 14yds. 1ft. 2in.

16. If it take 5 men to make 150 pair of shoes in 20 days, how many men can make 1350 pair in 60 days?

Ans. 15 men.

17. If a footman travel 240 miles in 12 days, when the days are 12 hours long; how many days will he require to travel 720 miles, when the days are 16 hours long?

Ans. 27 days.

18. If 4 men receive \$24 for 6 days' work, how much will 8 men receive for 12 days' work?

Ans. \$96.

19. If 9 persons in a family spend \$1512 in 1 year (or 12 mo.), how much will 3 of the same persons spend in 4 months?

Ans. \$168.

20. A regiment of soldiers, consisting of 800 men, are to be clothed, each suit containing 4 $\frac{1}{2}$  yds. of cloth, which is 1 $\frac{3}{4}$  yd. wide, and lined with flannel  $\frac{3}{4}$  yd. wide; how many yards of flannel will be sufficient to line all the suits?

Ans. 8633 yds. 1 qr. 1 $\frac{1}{2}$  na.

## PRACTICE.

Practice is a short method of doing all sums in the Single Rule of Three, that have one for their first term, and is of great use among merchants.

It may be proved by Compound Multiplication, or by the Single Rule of Three.

*Questions to prepare the learner for this rule.*

1. What will 50 yards of tape cost at  $\frac{1}{2}$  of a cent per yard?
2. What will 40 pounds of beef come to at  $\frac{1}{4}$  of a cent per pound?
3. What will 100 figs come to at  $\frac{3}{4}$  of a cent a piece?
4. How many pence will 40 peaches come to at one farthing a piece?
5. How many shillings and pence will 52 peaches come to at one farthing a piece?

## PRACTICE TABLE, OR TABLE OF ALIQUOT PARTS.

cts.	dols.	s.	d.	£.	d.	s.	
50 =	$\frac{1}{2}$	10 0 =	$\frac{1}{2}$	1	1 =	$\frac{1}{2}$	of a shilling.
25 =	$\frac{1}{4}$	6 8 =	$\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$\frac{1}{6}$	
20 =	$\frac{1}{5}$	5 0 =	$\frac{1}{4}$	2	2	$\frac{1}{6}$	
12 $\frac{1}{2}$ =	$\frac{1}{8}$	4 0 =	$\frac{1}{5}$	3	3	$\frac{1}{4}$	
6 $\frac{1}{2}$ =	$\frac{1}{16}$	3 4 =	$\frac{1}{6}$	4	4	$\frac{1}{3}$	
5 =	$\frac{1}{20}$	2 6 =	$\frac{1}{10}$	6	6	$\frac{1}{2}$	
4 =	$\frac{1}{25}$	1 8 =	$\frac{1}{12}$	qrs.	2 or	56 =	of a pound.
1 m.	cts.	1 0 =	$\frac{1}{20}$	1	1	28	
5 =	$\frac{1}{2}$	qr.	d.			16	
2 =	$\frac{1}{5}$	2	$\frac{1}{2}$	d.		14	
1 =	$\frac{1}{10}$	1	$\frac{1}{4}$			8	
						7	

## CASE I.

When the price of one yard, pound, &c. is in farthings.

## RULE.

Divide by the aliquot parts of a penny, and the answer will be in pence, which reduce to shillings, pounds, &c.

## EXAMPLES.

1. What is the value of 380 at one farthing each?      2. What is the value of 744 at 3 farthings each?

$$\begin{array}{r}
 1 \mid \frac{1}{4} \mid 380 \\
 \hline
 12 \overline{) 95} \\
 \hline
 7\text{s. } 11\text{d. Ans.}
 \end{array}$$

$$\begin{array}{r}
 2 \mid \frac{1}{3} \mid 744 \\
 \hline
 1 \mid \frac{1}{2} \mid 372 \\
 \hline
 186 \\
 \hline
 12 \overline{) 558} \\
 \hline
 20 \overline{) 46-6} \\
 \hline
 \text{£}2 \text{ } 6\text{s. } 6\text{d. Ans.}
 \end{array}$$

3. What is the value of 460 at 2 farthings each?

$$\begin{array}{r}
 2 \mid \frac{1}{2} \mid 460 \\
 \hline
 12 \overline{) 230} \text{ pence.} \\
 \hline
 19\text{s. } 2\text{d. Answer.}
 \end{array}$$

4. What is the worth of 298 at  $\frac{1}{4}$ d.?      Ans. 6s. 2 $\frac{1}{2}$ d.  
 5. What is the worth of 586 at  $\frac{1}{2}$ d.?      Ans. £1. 4s. 5d.  
 6. What is the worth of 964 at  $\frac{3}{4}$ d.?      Ans. £3. 0s. 3d.



## CASE II.

*When the price is any number of pence less than 12.*

## RULE.

Divide by the aliquot parts of a shilling, and the answer will be in shillings, which may be reduced to pounds.

## EXAMPLES.

I.		II.	
1	$\frac{1}{12}$	2	$\frac{1}{6}$
	672 at 1d.		444 at 2d.
	<u>20)56</u>		<u>20)74</u>
	£2. 16s. Ans.		£3 14s. Ans.
		£ s. d.	
3.	What is the value of 237 at 3d.?	Ans.	2 19 3
4.	What is the value of 594 at 4d.?	Ans.	9 18 0
5.	What is the value of 868 at 6d.?	Ans.	21 14 0
6.	What is the value of 988 at 5d.?	Ans.	20 11 8
7.	What is the value of 1049 at 8d.?	Ans.	34 19 4
8.	What is the value of 1294 at 10d.?	Ans.	53 18 4

## CASE III.

*When the price in pence exceeds the number of 12.*

## RULE.

Consider the number given in the sum as containing so many shillings. Then divide by such aliquot parts as may be formed by the pence over a shilling, adding the product to the sum. The answer will be in shillings.

## EXAMPLES.

I.	
$1\frac{1}{2}$	$\frac{1}{8}$
	600 at $13\frac{1}{2}$ d.
	<u>75</u>
	20
	<u>675</u>
	£33 15s. Answer.

*Note.*—In this example, I consider the sum as 600 shillings. Then, as the given price is  $13\frac{1}{2}$ d. over a shilling, which makes  $\frac{1}{8}$  of a shilling, I divide the sum by 8, and add the quotient to the given sum; which makes 675 shillings, or £33 15s.

2. What is the worth of 450 at 14d.? Ans. £26. 5s.

3. What is the worth of 570 at 16d.? Ans. £38. 0s.

### CASE IV.

*When the price is any number of shillings under 20.*

#### RULE.

Divide by the aliquot parts of a pound, and the answer will be in pounds. Or, consider the sum as being so many shillings, then multiply the sum by the number of shillings in the price. The product will be the answer in shillings; which reduce to pounds.

5s.	I. $\frac{1}{4}$	1296 at 5s.	II. 12×	723 at 12s
	4	£324 Ans.	12×	
			20)8676	

£433. 16s. Answer.

The second example is done by the second method, which is thought by many to be the easier way.

- |  | £     | s. |
|--|-------|----|
| 3. What is the value of 1128 at 3s.? Ans.  | 169.  | 4  |
| 4. What is the value of 889 at 4s.? Ans.   | 177.  | 16 |
| 5. What is the value of 1616 at 9s.? Ans.  | 727.  | 4  |
| 6. What is the value of 2868 at 18s.? Ans. | 2581. | 4  |

### CASE V.

*When the price is in pounds, shillings and pence.*

#### RULE.

Multiply the sum or quantity by the number of pounds in the price, then divide the aliquot parts of shillings and pence, and add the quotients to the product—their sum will be the answer.

I. EXAMPLES.			II.		
10	$\frac{1}{2}$	448 at £4 10s. 6d. 4	$\frac{1}{5}$		5678 at £7 4s. 9d
	4	1792	7		39746
	$\frac{1}{20}$	224	$\frac{1}{8}$		1135 12
	11 4	11 4	$\frac{1}{3}$		141 19
		£2027. 4s. Ans.			70 19 6
					£41094. 10s. 6d.

*Note.*—In the second example, after multiplying the sum by the number of pounds, as 4s. is  $\frac{1}{2}$  of a pound, I divide by 5, which gives 1135 pounds in the quotient; and leaving a remainder of 3 pounds, which reduced to shillings and divided by 5 give 12s. Then, as 6 and 3 make 9, the number of pence, and as 6d. is  $\frac{1}{8}$  of 4s., I divide the quotient by 8, which gives 141 pounds with a remainder of 7; this being reduced to shillings, and the 12 shillings above added to it make 152, which divided still by the 8 give 19 shillings. And as 3 is  $\frac{1}{2}$  of 6, or its aliquot part, I divide the last quotient by 2—This gives 70 pounds and a remainder of 1, which is 20 shillings; and adding it with 19 shillings above, the amount is 39 shillings. This divided by the 2 gives 19 shillings and a remainder of 1 shilling, or 12 pence; which divided still by the 2, makes 6d. And thus the answer is obtained.

3. What is the amount of 288 at £5. 10s. 4d.?

Ans. £1588. 16s.

4. What is the amount of 642 at £9. 4s. 6d.?

Ans. £5922. 9s.

5. What is the amount of 734 at £12. 2s. 8d.?

Ans. £8905. 17s. 4d.

#### CASE VI.

*When the quantity consists of different denominations, and the price is in pounds, shillings, &c.*

#### RULE.

Multiply the price of the highest denomination given, by the whole of the highest denomination, then divide by aliquot parts of each of the lower denominations in the sum. Add the results together, and their sum will be the answer.

#### EXAMPLES.

I.		£	s.	d.
3 cwt. 2 qrs. 14 lbs.		at 4	6	2 per cwt
2 qrs. are $\frac{1}{2}$ of a cwt..	$\frac{1}{2}$			3
			12	18 6
14 lbs. are $\frac{1}{4}$ of 2 qrs.	$\frac{1}{4}$		2	3 1
			10	9 $\frac{1}{4}$
		£15. 12s. 4 $\frac{1}{4}$ d. Ans.		

3. 4 cwt. 3 qrs. 12 lbs. at £8. 4s. 4d. per cwt.  
Ans. £39. 18s. 2d.
3. 5 cwt. 3 qrs. 4 lbs. at £9. 6s. 8d. per cwt.  
Ans. £54. 0. 0.
4. 7 cwt. 0 qr. 14 lbs. at £2. 3s. 4d. per cwt.  
Ans. £15. 8s. 9d.
5. 8 cwt. 3 qrs. 24 lbs. at £1. 2s. 3d. per cwt.  
Ans. £9. 19s. 5½d.
6. 9 cwt. 1 qr. 18 lbs. at £3. 10s. 10d. per cwt.  
Ans. £33. 6s. 7d.
7. 10 cwt. 2 qrs. 10 lbs. at £4. 4s. 6d. per cwt.  
Ans. £44. 14s. 9½d.

- Q. 1. What is practice?  
2. Wherein is it particularly useful?  
3. Repeat the table of aliquot parts.  
4. How many cases are there in pounds, shillings, &c.?  
5. Repeat the rule of each different case.  
6. How are sums in practice proved?

## FELLOWSHIP.

Fellowship is an easy rule by which merchants or other persons in company, are enabled to make a just division of the gain or loss in proportion to each person's share. Sums in Fellowship are generally done by the Rule of Three.

### CASE I.

*When the several shares are considered without regard to time.*

#### RULE.

As the sum of all the stock is to each person's particular share of the stock, so is the sum of all the gain or loss, to the gain or loss of each person.

#### PROOF.

Add together all the shares of gain or loss, and if it be right, the sum will be equal to the whole gain or loss.

## EXAMPLES.

1. A and B purchase certain goods amounting to \$580, of which A pays \$350 and B \$230. They gain 262;—what is each man's share of the gain?

A \$350

B \$230

—A's share. gain. \$ cts.

580 : 350 : : 262 : 158.10 $\frac{2}{3}$  A's gain.

B's share. gain. \$ cts.

Then, as 580 : 230 : : 262 : 103.89 $\frac{3}{8}$  B's gain.

2. A, B and C formed a company. A put in \$40, B 60 and C 80. They gained \$72:—what was each man's share? Ans. A gained \$16, B 24 and C 32.

3. A, B and C lose a quantity of property worth \$2400; of which A owned  $\frac{1}{4}$ , B  $\frac{1}{3}$ , and the remainder to C; what does each lose?

Ans. A loses \$600, B 800 and C 1000.

4. Three persons entered into partnership in trade. The first put in 250 dollars; the second put in 350 dollars; and the third put in 500 dollars; and in 12 months, they found, by examining their books, that they had gained 460 dollars; how must the gain be divided between them, so that each may have his due proportion?

Answer.  $\left\{ \begin{array}{l} \text{A's share, } \$104.54, 545 + \\ \text{B's share, } 146.36, 363 + \\ \text{C's share, } 209.09, 09 + \end{array} \right.$

5. A and B purchase goods worth \$80, of which A pays 30 and B 50. They gain \$20;—what is the gain of each? Ans. A's gain is \$7.50 and B's 12.50.

6. Four men formed a capital of \$3200. They gained in a certain time \$6560. A's stock was \$560, B's 1040, C's 1200 and D's 400. What did each gain?

Ans. A gained \$1148, B 2132, C 2460 and D 820.

## CASE II.

*When the different stocks in company are considered in relation to time.*

## RULE.

Multiply each man's stock by the time it has been a part of the whole stock; then, as the sum of the products is to either single product, so is the whole sum of gain or loss to the gain or loss of each man.

## EXAMPLES.

1. A, B and C hold a pasture in common, for which they pay \$40 per annum. A put in 9 cows for five weeks; B, 12 cows for 7 weeks; and C, 8 cows for 16 weeks.—What must each man pay for the rent?

$$9 \times 5 = 45$$

$$12 \times 7 = 84$$

$$8 \times 16 = 128$$

$$\text{As } 257 : 45 : : 40 : 7\frac{1}{257} \text{ A's part.}$$

$$\text{As } 257 : 84 : : 40 : 13\frac{19}{257} \text{ B's part.}$$

$$\text{As } 257 : 128 : : 40 : 19\frac{237}{257} \text{ C's part.}$$

2. A with a capital of £1000., entered into business on the first of January. On the first of March following he took in B as a partner, who brought with him a capital of £1500.; and three months after they are joined by C, with a capital of £2800. At the end of the year they find they have gained £1776. 10s. How must it be divided among them?

Ans. A's part will be £457. 9s. 4½d.

B's part will be £571. 16s. 8½d.

C's part will be £747. 3s. 11½d.

Q. 1. What is Fellowship?

2. By what rule are sums in Fellowship usually done?

3. How do you proceed when the shares are considered without regard to time?

4. How do you proceed when the shares are considered in relation to time?

5. How are sums in Fellowship proved?



## TARE AND TRET.

Tare and Tret are certain allowances made by merchants in selling their goods by weight.

*Tare* is an allowance made for the weight of the barrel, bag, &c., that contains the article or commodity bought.

*Tret* is an allowance of 4 lbs. in every 104 lbs. for waste, dust, &c.

*Gross weight* is the weight of the goods, together with the barrel, box, or whatever contains them. When the tare is deducted from the gross, what remains is called *suttle*.

*Neat weight* is the weight of articles after all allowances are deducted.

**CASE I.**

*When the tare is so much in the whole gross weight.*

**RULE.**

Subtract the tare from the quantity—the remainder will be the neat weight.

**EXAMPLES.**

In 6 hhd. of sugar, each weighing 9 cwt. 2 qrs. 10 lbs. gross, tare 25 lbs. per hhd. how much neat weight?

cwt.	qr.	lb.	cwt.	qrs.	lbs.
25 × 6 = 1	1	10 tare	9	2	10
					6 ×

57	2	4 gross.
1	1	10 tare

56	0	22 Ans.
----	---	---------

2. What is the neat weight of 456 cwt. 1 qr. 19 lbs. of tobacco, tare in the whole 15 cwt. 2 qrs. 13 lbs?

Ans. 440 cwt. 3 qrs. 6 lbs.

3 What is the neat weight of 5 casks of sugar, the gross weight and tare as follows?

		cwt.	qrs.	lbs.		qrs.	lb.
No. 1.	Gross	4	2	14	Tare	1	5
2.	—	3	0	17	—	1	1
3.	—	5	3	10	—	2	11
4.	—	6	1	16	—	2	27
5.	—	3	2	18	—	1	3

Ans. 21 cwt. 2 qrs.

**CASE II.**

*When the tare is at so much per cwt.*

**RULE.**

Divide the gross weight by the aliquot parts of a cwt then subtract the quotient from the gross, and the remainder will be the neat weight.

## EXAMPLES.

1. In 129 cwt. 3 qrs. 16 lbs. gross, tare 14 lbs. per cwt. what neat weight?

$$\begin{array}{r|l}
 14 \text{ lbs.} & \frac{1}{8} \\
 \hline
 129 & 3 & 16 \text{ gross.} \\
 16 & 0 & 26\frac{1}{2} \\
 \hline
 113 & 2 & 17\frac{1}{2} \text{ Answer.}
 \end{array}$$

2. In 97 cwt. 1 qr. 7 lbs. gross, tare 20 lbs. per cwt. what neat weight?

Ans. 79 cwt. 1 qr.  $20\frac{3}{4}$  lbs.

2. What is the neat weight of 35 kegs of raisins, gross weight 37 cwt. 1 qr. 20 lb.;—tare per cwt. 14 lbs.?

Ans. 32 cwt. 3 qrs.

3. What is the neat weight of 6 hogsheads of sugar, each weighing 8 cwt. 2 qrs. 14 lbs. gross; tare 16 lbs. per cwt.?

Ans. 44 cwt. 1 qr. 12 lbs.

*Note.*—When the tare per cwt. is not an aliquot part, the tare may be found by the Rule of Three, thus—As 112 is to the number of pounds gross, so is the rate per cwt., to the tare required.

4. What is the neat weight of 38 cwt. 0 qr. 4 lbs. tare, at 11 lbs. per cwt.

$$\begin{array}{rcl}
 \text{cwt.} & \text{qr.} & \text{lbs.} \\
 38 & 0 & 4 = 4260 \text{ pounds.}
 \end{array}$$

$$\begin{array}{rcl}
 \text{lbs.} & \text{lbs.} & \text{lbs.}
 \end{array}$$

Then, as 112 : 4260 : : 11 :  $418\frac{44}{112}$  Answer.

$$\begin{array}{r}
 4260 \\
 418\frac{44}{112} \\
 \hline
 \end{array}$$

$$\begin{array}{rcl}
 & \text{cwt.} & \text{qr.} & \text{lbs.} \\
 3841\frac{68}{112} \text{ neat} = & 34 & 1 & 5\frac{68}{112} \text{ Answer.}
 \end{array}$$

## CASE III.

*When tare and tret are allowed.*

## RULE.

Find the tare according to the preceding rules, subtract it from the gross, and the remainder will be *suttle*; then divide the *suttle* by 26, and the product will be the *tret*, which subtract from the *suttle*—the remainder will be the *neat*.

*Note.*—As 4 pounds on the 104 lbs. is the customary allowance for *tret*, we divide by 26, because 4 is  $\frac{1}{26}$  of 104.



## EXAMPLES.

1. In 247 cwt. 2 qrs. 15 lbs. gross, tare 28 lbs. per cwt. and tret 4 lbs. for every 104 lbs. how much neat?

	cwt.	qr.	lb.	oz.	
28 lbs. = $\frac{1}{4}$ cwt.	247	2	15		
	61	3	17	12	tare subtract.
<hr/>					
4 lbs. = $\frac{1}{26}$ of 104	185	2	25	4	
	7	0	16	0	
<hr/>					
Ans.	178	2	9	4	neat.

2. In 9 cwt. 1 qr. 10 lbs. gross, tare 28 lbs. per cwt. and tret 4 lbs. for every 104 lbs. how much neat?

Ans. 6 cwt. 2 qrs. 26 lbs.

3. A merchant purchased 4 hhds. of tobacco, weighing as follows:—The first 5 cwt. 1 qr. 12 lbs. gross, tare 65 lbs. per hhd.; the 2d. 3 cwt. 0 qr. 19 lbs. gross, tare 75 lbs.; the 3d. 6 cwt. 3 qrs. gross, tare 49 lbs.; the 4th 4 cwt. 2 qrs. 9 lbs. gross, tare 35 lbs. and allowing tret to each at the rate of 4 lbs. for every 104 lbs. What was the neat weight of the whole?

Ans. 17 cwt. 0 qr. 19 lbs. 2 oz.

*Exercises under the foregoing rules.*

1. There are 24 hogsheads of tobacco; each hoghead weighs 6 cwt. 2 qrs 17 lbs. gross;—tare in all, 17 cwt. 3 qrs. 27 lbs. How much will the tobacco amount to at £1. 10s. 6d. per cwt. Ans. £216. 0s. 4½d.

2. Bought 5 bags of coffee, each of which weighed 95 lbs. gross; tare in the whole 10lbs. How much did it amount to, at 25 cents per pound? Ans. \$116.25.

3. What is the value of 10 casks of alum; the whole weighing 33 cwt. 2 qrs. 15 lbs. gross; tare 15 lbs. per cask; price, 23s. 4d. per cwt.? Ans. £37. 13s. 6½d.

4. A farmer sent a load of hay to market, which with the cart, weighed 29 cwt. 3 qrs. 16 lbs.; the weight of the cart was 10¾ cwt.; what did the hay come to, at \$15 a ton? Ans. \$14.357+.

5. A merchant bought sugar in a hogshead, both of which weighed 8 cwt. 15 lbs.; the hogshead alone weighed 1 cwt. 1 qr.; what was the cost of the sugar, at  $11\frac{1}{4}$  cents a pound?

Ans. \$86.73 $\frac{3}{4}$ .

- Q. 1. What do you understand by Tare and Tret?
2. What is tare?
  3. What is tret?
  4. What is gross weight?
  5. What is neat weight?
  6. What is calleduttle?

## SIMPLE INTEREST.

Interest is a premium paid for the use of money. In calculating interest on money, four things are necessary to be considered, viz. the principal, the time, rate per cent, and amount.

The *principal* is the money lent for which interest is to be received.

The *rate per cent.* per annum (by the year) is the interest for 100 dollars or 100 pounds for one year.

The *time* is the number of years, months, or days, for which interest is to be calculated.

The *amount* is the sum of the principal and interest, when added together.

*Questions to prepare the learner for this rule.*

1. If you give \$6 for the use of \$100 for a year; how much must you give for the use of \$50?

2. If you give \$6 for the use of \$100 for a year; how much must you give for the use of it for six months?—How much for three months?—How much for 4 months? How much for 8 months?—How much for 9 months?

3. If the interest of \$200 be one dollar for a month; how much will it be for 15 days?—How much for 10 days?—How much for 20 days?

## CASE I.

*When the time is one year, and the rate per cent. is any number of dollars, pounds, &c.*

## RULE.

Multiply the principal by the rate per cent. divide the product by 100, and the quotient will be the interest for one year.

## EXAMPLES.

1. What is the interest of 328 dollars for one year at 6 per cent.?

328  
6

In this example, as cutting off the two right hand figures is the same as dividing by 100, the division is omitted.

Ans. \$19.68 cts.

2. What is the interest of \$9876 for one year at 6 per cent.?

6

\$592.56 cts. Answer.

When the sum is in pounds, if there be a remainder after dividing, or after cutting off the two right hand figures, the remainder, or figures cut off must be reduced to shillings; and if there be still a remainder after dividing the shillings, it must be reduced to pence, &c.

3. What is the interest of £573. 13s. 9½d. at 6 per cent. per annum?

£573. 13s. 9½d.  
6

£34.42 2 9  
20  
8.42  
12

*Note.*—When the interest is for more than one year, multiply the interest for one year by the number of years. To obtain the *amount*, the interest must be added to the principal.

5.13

Ans. £34. 8s. 5d.

4. What is the interest of £40. 19s. 11d. 3 qrs. for one year, at 6 per cent. per annum?

Ans. £2. 9s. 2d. 1qr.

5. What is the interest of 87 dollars for one year, at 6 per cent. per annum? Ans. \$5.22.

6. What is the interest of 143 dollars for one year, at 7 per cent. per annum? Ans. \$10.01.

When the rate per cent. consists of a whole number and a fraction, as  $6\frac{1}{4}$ ,  $6\frac{1}{2}$ , or  $6\frac{3}{4}$ , multiply the principal by the whole number, to the product add  $\frac{1}{4}$ , or  $\frac{1}{2}$ , as the case may be, of the principal and then divide by 100, or cut off the two right hand figures as before.

7. What is the interest of 228 dollars for one year, at  $6\frac{1}{4}$  per cent per annum?

\$228

$6\frac{1}{4}$

---

1368

57

---

\$14|25 cts Answer.

When the principal consists of dollars and cents, multiply by the rate per cent. without any reference to the separating point; then from the product cut off the first right hand figure as a fraction or remainder, the next figure will be mills, the two next cents, and the other figures, that is, those on the left of the cents, will be dollars.

8. What is the interest of \$98.79 for one year, at 6 per cent. per annum? 6

---

5|92|7|4 fraction

Ans. \$5.92c. 7m.

9. What is the interest of 432 dollars 73 cents for 4 years, at 6 per cent. per annum?

\$432.73

6 rate per cent.

---

259638

4 number of years.

---

103|85|5|2 frac. Ans. \$103.85c. 5m.

10. What is the interest of \$8420.82 for three years, at 8 per cent. per annum? Ans. \$2020 99c 6m.

11. What is the interest and amount of \$7462.13½ for four years, at 7 per cent per annum?

Ans. Interest, \$2089.39c. 7m. Am't. \$9551.53c. 2m.

## CASE II.

*To find the interest when the given time is months or days.*

## RULE.

Find the interest for one year, then say—as one year is to the given time, so is the interest of the sum for one year, to the interest for the time required. Or, instead of the Rule of Three, it may be done by Practice, thus: For the number of months, take aliquot parts of a year; and for days, the aliquot parts of 30.\*

## EXAMPLES.

1. What is the interest of \$98.50 for 9 months and 18 days, at 6 per cent. per annum?

\$98.50

6

\$5.91|00 for one year.

year. mo. days.      \$ cts.    \$ cts. m.  
Then, as 1 : 9 18 : : 5 91 : 4 72 8 Ans.

In this sum, the year is reduced to 360 days, the 9 months and 18 days to 288 days, and the third term stands as 591 cents.

The same is done by Practice, thus—

		\$98.50
		6
mo.		
6, ½ of a year.	½	5.91.0 0
3, ½ of 6 mo.	½	2.95.5
15d. ½ of 3 mo.	½	1.47.7½
3, ⅓ of 15 ds.	⅓	24.6¼
		4.9¼

Ans. \$4.72.8

\*In these calculations, a year is reckoned at 360 days, and a month at 30 days.

2. What is the interest of \$120.60 for one year and three months, at 6 per cent. per annum?

Ans. \$9.04c. 5m.

3. What is the interest on \$187.06 $\frac{1}{4}$  for 10 months, at 6 per cent per annum?

Ans. \$9.35c. 3m.

4. What is the interest and amount of 640 dollars for 4 years and 7 months, at 5 per cent. per annum?

Ans. \$146.66 $\frac{2}{3}$  interest. Am't. \$786.66 $\frac{2}{3}$ .

5. What is the interest of \$300 for 4 years, 4 months, and 20 days, at 8 $\frac{1}{2}$  per cent. per annum?

Ans. \$111.91 $\frac{1}{2}$ .

6. What is the interest of \$5420 for 17 months, at 4 per cent per annum?

Ans. \$307.13 $\frac{1}{3}$ .

7. What is the interest of \$7200 for 14 months, at 6 per cent. per annum?

Ans. \$504.

8. What is the interest of \$8050.87 $\frac{1}{2}$  for 3 years and 11 months, at 6 per cent. per annum?

Ans. \$1891.95c. 5m.

9. What is the interest of \$948.62 $\frac{1}{2}$  for 8 months, at 8 per cent. per annum?

Ans. \$50.59c. 3m.

10. What is the interest of £421. 16s. 9d. for 2 years and 8 months, at 5 per cent. per annum?

Ans. £56. 4s. 10 $\frac{1}{2}$ d.

11. What is the interest of 580 pounds for 5 years, 2 months and 10 days, at 7 per cent per annum?

Ans. £210. 17s. 10 $\frac{1}{2}$ d.

12. What is the interest of \$36 for 1 month, at 8 per cent. per annum?

Ans. 24 cents.

*When the rate is 6 per cent. another method of finding the interest for any number of months, is, to multiply the principal by half the number of months and divide the product by 100 (or cut off the two right hand figures as before.)*

#### EXAMPLES.

1. What is the interest of \$1500 for 4 months?

\$1500

2 half the number of months.

30|00

Ans. \$30.

2. What is the interest of \$7656 for 3 years and 4 months?

\$7656

20 half the number of months.

1531|20

Ans. \$1531.20.

3. What is the interest of \$230.25 for 8 months?

Ans. \$9.21.

4. What is the interest of \$750 for 9 months?

Ans. \$33.75.

5. What is the interest of \$967.64 for 28 months?

Ans. \$135.47.

*The interest for any number of days, at 6 per cent. can be found by multiplying the dollars by the number of days, and dividing the product by 60: the answer will be in cents. If the principal consist of dollars and cents, cut off the two right hand figures. Bank interest is reckoned by this rule.*

1. What is the interest of \$1542 for 90 days? And of \$754.54 for 60 days?

\$1542

90

6|0)13878|0

2313 Ans. \$23.13.

\$754.54

60

6|0)452724|0

754|54 Ans. \$7.54.

2. What is the interest of \$3084 for 30 days at 6 per cent. per annum?

Ans. \$15.42.

3. What is the interest of \$2324 for 54 days, at 6 per cent. per annum?

Ans. \$20.91.

4. What is the interest of \$281.75 for 93 days, at 6 per cent. per annum?

Ans. \$4.36.

### CASE III.

*The amount, time, and rate per cent. given to find the principal.*

#### RULE.

Find the amount of 100 dollars at the rate and time given; then say, as the amount of 100 dollars, is to the amount given, so are 100 dollars to the principal required.

## EXAMPLES.

1. What principal at interest for two years, at 6 per cent. per annum, will amount to \$134.40?

$$\begin{array}{r}
 \$100 \\
 6 \\
 \hline
 6.00 \\
 2 \\
 \hline
 12.00 \\
 100.00 \\
 \hline
 \end{array}$$

\$112 amount of 100 for two years.

dolls. \$ cts. dolls. dolls.

Then, as 112 : 134.40 :: 100 - 120 Ans.

2. What principal at interest for 5 years, at 6 per cent. will amount to \$780? Ans. \$600.

3. What principal at interest for 4 years and 3 months, at 6 per cent. will amount to \$1192.25? Ans. \$950.

## CASE IV.

*To find the rate per cent. when the amount, time and principal are given.*

## RULE.

Take the principal from the amount, the remainder will be the interest for the given time; then, as the principal is to one hundred dollars, so is the interest of the principal for the given time, to the interest of 100 dollars for the same time. Divide the interest of 100 dollars thus found, by the time, and the quotient will be the rate per cent.

## EXAMPLES.

1. At what rate per cent. will \$500 amount to \$650 in three years?

650 Amount.

500 Principal.

150 Interest for the time.

D. D. D. D.

As 500 : 100 :: 150 . 30 Interest of 100.

Then divide by the time 3)30(10 Ans. per cent.

30



2. At what rate per cent. per annum will \$1850 double in 5 years?      Ans. 20 per cent.

3. At what rate per cent. per annum, will 600 dollars amount to \$856.50 in 9 years and 6 months?

Ans.  $4\frac{1}{2}$  per cent.

### CASE V.

*To find the time when the principal, amount, and rate per cent. are given.*

#### RULE.

Find the interest of the principal for one year; find the interest of the principal for the whole time, by subtracting the principal from the amount; then divide the whole interest by the interest for one year—the quotient will show the time required.

#### EXAMPLES.

1. In what time will \$800 amount to \$1000 at 5 per cent. per annum?

$$\begin{array}{r} 800 \\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 1000 \\ 800 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Then, } 4|0)20|0 \\ 5 \end{array}$$

$$\$40|00$$

$$200 \text{ Whole In't. Ans. 5 years.}$$

2. In what time will \$80 amount to \$182.40 at 8 per cent. per annum?      Ans. 16 years.

3. In what time will \$600 amount to \$798 at 6 per cent. per annum?      Ans.  $5\frac{1}{2}$  years.

### COMPOUND INTEREST.

Compound Interest is that which arises from the interest being added to the principal, and becoming a part of the principal, at each time of payment.

#### RULE.

Find the *amount* of the principal, for the time of the first payment, by Simple Interest; this amount, containing the *principal and interest* for the first year, will be the *principal* for the second year; and the *amount* of this *principal*, which consists of the *principal and interest* for the second year, will be the *principal* for the third year, and so on, for any number of years. From the last amount; subtract the given principal, and the remainder will be the compound interest.

## EXAMPLES.

1. What is the compound interest of \$8000 for two years, at 6 per cent per annum?

\$8000

6

Interest for the first year 480|00

Principal 8000

Amount 8480

6

In't. for the second year 508|80

Principal 8480.00

8988.80

Subtract 8000.00

\$988.80c. Answer.

2. What is the compound interest of \$554 for 3 years, at 8 per cent. per annum? Ans. \$143.88.

3. What is the compound interest of \$744 for 2 years, at 7 percent. per annum? Ans. \$107.80c. 5m

4. What is the compound interest of \$50 for 8 years, at 8 per cent. per annum? Ans. \$42.54c. 6m

5. What is the compound interest of £48. 5s. for 3 years, at 6 per cent. per annum? Ans. £9. 4s. 3½d.

*In computing Interest on Notes.*

When a settlement is made within a short time from the date or commencement of interest, it is generally the custom to proceed according to the following

**RULE.**

*Find the amount of the principal, from the time the interest commenced to the time of settlement, and likewise the amount of each payment, from the time it was paid to the time of settlement; then deduct the amount of the several payments from the amount of the principal.*

*Exercises for the Slate.*

1. For value received, I promise to pay Rufus Stanly, or order, Three Hundred Dollars, with interest. April 1, 1825.

\$300.

PETER MOSELY.

On this note were the following endorsements:—

October 1, 1825, received \$100 }  
 April 16, 1826, . . . \$ 50 }  
 December 1, 1827, . . . \$120 }

What was due April 1, 1828? Ans. \$60.73.

#### CALCULATION.

*The first principal on interest from April 1, 1825, - \$300.00*  
*Interest to April 1, 1828, (36 mo.), - - - 54.00*

<i>Amount of principal</i>	-	-	-	-	<i>\$354.00</i>
<i>First payment, Oct. 1, 1825,</i>	-	-	-	<i>\$100.00</i>	
<i>Interest to April 1, 1828, (30 mo.)</i>	-	-	-	<i>15.00</i>	
<i>Second payment, April 16, 1826,</i>	-	-	-	<i>50.00</i>	
<i>Interest to April 1, 1828, (23½ mo.)</i>	-	-	-	<i>5.87</i>	
<i>Third payment, Dec. 1, 1827,</i>	-	-	-	<i>120.00</i>	
<i>Interest to April 1, 1828, (4 mo.)</i>	-	-	-	<i>2.40</i>	

*Amount of payments deducted - - \$293.27*

*Remains due, April 1, 1828, - - \$60.73*

2. For value received, I promise to pay Peter Trusty, or order,  
 Five Hundred Dollars, with interest. July 1, 1825.

\$500.

JAMES CARELESS.

#### ENDORSEMENTS.

July 16, 1826, received \$200 }  
 Jan. 1, 1827, - - - \$ 40 }  
 March 16, 1827, - - - \$230 }

What remained due July 16, 1828? Ans. \$75.15.

#### RULE IN SOME OF THE UNITED STATES.

*Compute the interest on the principal sum to the first time when a payment was made, which, either alone, or together with the preceding payments (if any,) exceeds the interest then due; add that interest to the principal, and from the sum subtract the payment, or the sum of the payments, made at that time, and the remainder will be a new principal, with which proceed as with the first principal, and so on, to the time of settlement.*

1. For value received, I promise to pay Jason Park, or order,  
 Six Hundred Dollars, with interest. March 1, 1822.

\$600.

STEPHEN STIMPSON.

#### ENDORSEMENTS.

May 1, 1823, received \$200 }  
 June 16, 1824, - \$ 80 }  
 Sept. 17, 1825, - \$ 12 }  
 Dec. 19, 1825, - \$ 15 }  
 March 1, 1826, - \$100 }  
 Oct. 16, 1827, - \$150 }

What was there due August 31, 1828? Ans. \$194.41.

The principal, \$600, on interest from March 1, 1822,	\$600.00
Interest to May 1, 1823, (14 mo.)	42.00
	<i>Amount,</i> \$642.00
Payment, May 1, 1823, a sum greater than the interest,	200.00
Due May 1, 1823, forming a new principal,	\$442.00
Interest on \$442, from May 1, 1823, to June 16, 1824, (13½ mo.)	29.83
	<i>Amount,</i> \$471.83
Payment, June 16, 1824, a sum greater than the interest then due,	80.00
Due June 16, 1824, forming a new principal,	\$391.83
Interest on \$391.83, from June 16, 1824, to March 1, 1826, (20½ mo.)	40.16
	<i>Amount,</i> \$431.99
Payment, a sum less than the interest then due,	\$ 12
Payment, a sum less than the interest then due,	\$ 15
Payment, a sum greater than the interest then due,	\$100
	<i>Amount,</i> \$127.00
Due March 1, 1826, forming a new principal,	\$304.99
Interest on \$304.99, from March 1, 1826, to Oct. 16, 1827, (19½ mo.)	29.73
	<i>Amount,</i> \$334.72
Payment, Oct. 16, 1827, a sum greater than the interest then due,	150.00
Due Oct. 16, 1827, forming a new principal,	\$184.72
Interest on \$184.72, from Oct. 16, 1827, to August 31, 1828, being the time of settlement, (10½ mo.)	9.69
Balance due Aug. 31, 1828,	\$194.41
2. For value received, I promise to pay Asher L. Smith, or order, Nine Hundred Dollars, with interest. June 16, 1820.	
\$900.	WILLIAM MORRIS.

## ENDORSEMENTS.

July 1, 1821, received	\$150
Sept. 16, 1822, - - - -	\$ 90
Dec. 10, 1824, - - - -	\$ 10
June 1, 1825, - - - -	\$ 20
Aug. 16, 1825, - - - -	\$200
March 1, 1827, - - - -	\$300

What remained due Sept. 1, 1828? . Ans. \$483.07.

Q. 1. What is Interest?

2. What are the four things considered in calculating interest?

3. What is the principal?—What is the rate per cent?  
What is the time?—What is the amount?

4. How do you proceed in the first case?
5. How do you proceed in pounds, shillings, &c.?
6. How do you proceed when the rate per cent. consists of a whole number and a fraction?
7. How do you proceed when the principal is in dollars and cents?
8. How do you calculate interest for more than a year?—How, when the time is in months?
9. What other method is there for calculating interest, besides the method of multiplying the sum by the rate per cent.?
10. How is bank interest reckoned?—What is the rule for casting it?
11. Do you understand all the cases and rules of interest?
12. What is Compound Interest?
13. Repeat the rule for calculating Compound Interest?

## INSURANCE, COMMISSION AND BROKERAGE.

Insurance, Commission and Brokerage, are premiums allowed to insurers, factors and brokers at a certain rate per cent.; and is obtained after the manner of the first case in Simple Interest.

### EXAMPLES.

1. What is the insurance of \$4500, at  $2\frac{1}{2}$  per cent.?

$$\begin{array}{r} 2\frac{1}{2} \\ \hline 9000 \\ 2250 \end{array}$$

\$112|50c. Answer.

2. What is the commission on a sale of goods amounting to \$1184 at 5 per cent.?

Ans. \$59.20.

3. What is the brokerage of \$987 at 3 per cent.?

Ans. \$29.61.

4. What is the commission on a sale of goods amounting to 4820 at  $4\frac{1}{2}$  per cent.?

\$216.90.

## DISCOUNT.

Discount is an allowance made for the payment of any sum of money before it becomes due, and is the difference between that sum, due some time hence, and its present worth.

## RULE.

As the amount of \$100 at the given rate and time is to \$100, so is the given sum or debt to the present worth. Subtract the present worth from the given sum, and the remainder will be the discount.

## EXAMPLES.

1. What is the present worth of \$500 due in 3 years, at 6 per cent. per annum?

$\begin{array}{r} \$100 \\ 6 \\ \hline 6 \overline{)100} \\ 18 \\ \hline 100 \\ \hline 118 \text{ amount of } \$100. \end{array}$	$\begin{array}{r} \$ \quad \$ \quad \$ \\ 118 : 100 : : 500 \\ 100 \\ \hline 118)50000 \text{ } \$ \text{ c.} \\ 472 \\ \hline 280 \\ 236 \\ \hline 440 \\ 354 \\ \hline 86.00 \overline{)72c.} \\ 826 \\ \hline 340 \\ 236 \\ \hline 104 \text{ remainder.} \end{array}$
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2. What is the present worth of \$350 payable in 6 months, discounting at 6 per cent. per annum?

Ans. \$339.80c. 5m.

3. What is the discount on \$1000 due in one year, at 6 per cent. per annum?

Ans. \$56.60c. 4m.

4. What is the present worth of £65 due in 15 months at 6 per cent. per annum?

Ans. £60. 9s. 3½d.

5. What sum will discharge a debt of \$1595 due after 5 months and 20 days at 6 per cent. per annum?

Ans. \$1541.32. 6m.

6. What is the present worth of \$426.55 at 6 per cent. per annum, due in 8 months? Ans. \$410.14c. 5m.

*Note.*—When discount is made without regard to time, it is found as the interest of the sum would be for one year.

## EQUATION.

Equation is the method for finding a time to pay at once, several debts due at different times.

### RULE.

Multiply each payment by the time at which it is due, and divide the sum of the products by the sum of all the payments—the quotient will be the time required.

### EXAMPLES.

1. A owes B \$480 to be paid in the following manner, viz: \$100 in 6 months, \$120 in 7 months, and \$260 in 10 months; what is the equated time for payment of the whole debt?

$$100 \times 6 = 600$$

$$120 \times 7 = 840$$

$$260 \times 10 = 2600$$

$$\begin{array}{r} 480 \qquad ) 4040 ( 8\frac{5}{12} \text{ months, Ans.} \\ \underline{3840} \end{array}$$

$$200$$

$$\frac{200}{480} = \frac{5}{12}$$

2. A owes B \$1100, of which 200 is to be paid in 3 months, 400 in 5 months, and 500 in 8 months—what is the equated time for payment of all? Ans. 6 months.

3. C is indebted to a merchant to the amount of \$2500; of which \$1000 is payable at the end of 4 months, \$800 in 8 months, and 700 in 12 months—when ought payment to be made, if all are paid together?

Ans. 7 months, 15 $\frac{1}{2}$  days.

## LOSS AND GAIN.

Loss and Gain is a rule by which persons in trade are able to discover their profit or loss; and to increase or lessen the prices of their goods so as to gain or lose on them to any given amount.

Questions in Loss and Gain are solved by the Rule of Three, or by Practice.

### EXAMPLES.

1. A merchant bought 100 yards of silk at 75 cents per yard, what will be his gain in the sale, if he sell it for 90 cents yer yard?

75 cents.

—                      *yard. yards. cts. dolls.*  
15 gain per yard. As 1 : 100 : : 15 : 15 Ans.

2. If a grocer buy 250 lbs. of tea, at \$225, and sell the whole at \$1.25 per lb. what will be his gain by the transaction?

Ans. \$87.50.

3. If a yard of calico cost 28 cents, and is sold for 31 cents, what is the gain on 293 yards?

Ans. \$8.79.

4. Bought 420 bushels of corn at 25 cents per bushel, and sold the same at 38 cents per bushel; what was the amount gained?

Ans. \$54.60.

5. A merchant bought 12 cwt. of coffee at 26 cents per lb. and afterwards was obliged to sell it at 20 cents per lb. what was his loss?

Ans. \$80.64.

6. If a merchant gain \$80 on \$560, what is that per cent.?

Ans. 14 $\frac{2}{7}$  per cent.

7. If a yard of velvet be bought for 16s. and sold again for 12s. what is the loss per cent.?

Ans. 25 per cent.

8. A merchant bought 2 hhds. of wine, containing 126 gals., at \$1.75 a gal. and retailed the same at \$2.12 $\frac{1}{2}$  a gal.: what did he gain in the whole?

Ans. \$47.25.

9. A merchant bought 2 pieces of broad-cloth, containing 56 yds., at \$4.75 a yard; but upon examination, found them damaged. He was, therefore, obliged to sell them for \$4.12 $\frac{1}{2}$  a yard; how much did he lose by the bargain?

Ans. \$35.



10. A gentleman purchased 1500 lbs. of coffee for \$172.50, how must he sell the same to gain \$32 by his bargain?  
Ans. 13 cts. 6 m. 33.

11. A merchant bought 250 bbls. of flour at \$4.50 a bbl.; how must he sell the same to gain \$55 by the bargain?  
Ans. \$4.72.

12. A lady purchased a quantity of millinery, for which she gave \$184; and sold the same for \$210; how much did she gain per cent.? Ans. 14.13+ per cent.

## INVOLUTION,

### OR THE RAISING OF POWERS.

The product of any number multiplied by itself any given number of times, is called its power, as in the following example.

Thus,  $2 \times 2 = 4$  the square, or second power of 2.

$2 \times 2 \times 2 = 8$  the cube, or third power of 2.

$2 \times 2 \times 2 \times 2 = 16$  the biquadrate, or fourth power of 2.\* Hence, 3 raised to the 4th power makes 81. The number which denotes a power is called the *index*, or *exponent* of that power.

When a power of a vulgar fraction is required, it is only necessary to raise, first the numerator, and then the denominator to the given power, and place the product of the one over the product of the other, thus, the 3d

power of  $\frac{2}{3}$   $\frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{8}{27}$

#### EXAMPLES.

1. What is the square of 4567? Ans. 20857489.

2. What is the cube of 567? Ans. 182284263.

3. What is the biquadrate of 67? Ans. 20151121.

4. What is the ninth power of 2? Ans. 512.

5. What is the cube of  $\frac{7}{8}$ ? Ans.  $\frac{343}{512}$ .

6. What is the cube or third power of .13? Ans. 002197

7. What is the sixth-power of 5.03?

Ans. 16196.005304479720.

\* Any given number is considered the first power of itself, and when multiplied by itself the product is the second power, &c:

# EVOLUTION,

## OR THE EXTRACTION OF ROOTS.

The root of a number, or power, is any number, which being multiplied by itself a certain number of times, will produce that power; and is called the square, cube, biquadrate root, &c. according to the power to which it belongs. Thus, 3 is the square root of 9, because when multiplied by itself, it produces 9; and 4 is the cube root of 64, because  $4 \times 4 \times 4 = 64$ , and so of any other number.

### THE SQUARE ROOT.

Extracting the square root of a number, is the taking a smaller number from a larger, and such as will, being multiplied by itself, produce the larger number.

#### RULE.

1. Separate the sum into periods of two figures each beginning at the right hand figure.
2. Seek the greatest square number in the left hand period; place the square, thus found, under that period, and the root of it in the quotient. Subtract the square number from the first period; to the remainder bring down the next period, and call that the resolvend.
3. Double the quotient, and place it on the left hand of the resolvend for a divisor. Seek how often the divisor is contained in the resolvend, omitting the units figure, and set the answer in the quotient, and also on the right hand side of the divisor. Then multiply the divisor, including the last added figure, by that figure, that is, by the figure last placed in the quotient; place the product under the resolvend, subtract it, and to the remainder bring down the next period, if there be any more, and proceed as already directed. If there be a remainder after the periods are all brought down, annex cyphers, two at a time, for decimals, and proceed till the root is obtained with sufficient exactness.

*Note.*—When a sum in the Square Root consists of whole numbers and decimals, point off the whole numbers as above directed, then point the decimal part,

commencing at the decimal point and forming periods of two figures each towards the right, observing when there is only one figure left for the last period, to add a cypher to the right of it, to make an even period.—When the sum consists entirely of decimals, separate the periods after the same manner. If it be required to extract the square root of a vulgar fraction, reduce it to its lowest terms; then extract the root of the numerator for the numerator of the answer, and the root of the denominator for the denominator of the answer. If the fraction be a surd, that is, a number whose root can never be exactly found, reduce it to a decimal, and then extract the root from it; and if the sum be a mixed number, the root may be obtained in the same way.

## PROOF.

Square the root, adding the remainder, (if any,) and the result will equal the given number.

## EXAMPLES.

1. What is the square root of 20857489?

$$\begin{array}{r} \text{Root.} \\ 20857489 \end{array} (4567 \text{ Answer.} \\ 16$$

$$\begin{array}{r} \text{divisor } 85 \overline{)185} \text{ resolvend.} \\ 425 \end{array}$$

$$\begin{array}{r} \text{divisor } 906 \overline{)3074} \text{ resolvend.} \\ 5436 \end{array}$$

$$\begin{array}{r} \text{divisor } 9127 \overline{)3889} \text{ resolvend.} \\ 63889 \end{array}$$

2. What is the square root of 294849?      Ans. 543.  
 3. What is the square root of 41242084?      Ans. 6422.  
 4. What is the square root of 17.3056?      Ans. 4.16.  
 5. What is the square root of .000729?      Ans. .027.  
 6. What is the square root of 5?      Ans. 2.23606.  
 7. What is the square root of  $\frac{27}{147}$ ?      Ans.  $\frac{3}{7}$ .  
 8. What is the square root of  $1\frac{2}{3}$ ?      Ans. 4.168333.

9. A general has an army of 7056 men; how many must he place on a side to form them into a compact square? Ans. 84.

10. If the area of a circle be 184.125, what is the side of a square that shall contain the same area?

Thus,  $\sqrt{184.125} = 13.569 + \text{Answer.}$

11. If a square piece of land contain 61 acres and 41 square poles, what is the length of one of its sides?

A. P.

Thus,  $61 \text{ } 41 = 9801$  square poles.

Then,  $\sqrt{9801} = 99$  rods, or poles in length, Answer.

12. There is a circle whose diameter is 4 inches; what is the diameter of a circle 3 times as large?

Thus,  $4 \times 4 = 16$ ; and  $16 \times 3 = 48$  and  $\sqrt{48} = 6.928 + \text{inches. Ans.}$

13. There is a circle whose diameter is 8 inches; what is the diameter of a circle which is only one fourth as large.  $8 \times 8 = 64$ ; and  $64 \div 4 = 16$ ; and  $\sqrt{16} = 4$  inches.

Ans. 4 inches.

*The square of the longest side of a right angled triangle, is equal to the sum of the squares of the other two sides; therefore, the difference of the squares of the longest side, and either of the other sides, is the square of the remaining side.*

14. The wall of a certain city is 20 feet in height, it is surrounded by a ditch 20 feet in breadth; what must be the length of a ladder, to reach from the outside of the ditch to the top of the wall? Ans.  $28\frac{1}{2}$  feet.

15. On the margin of a river 24 yards wide, stands a tree; from the top of which a line 36 yards long, will reach to the other side of the stream; what is the height of the tree? Ans.  $26.83 +$  yards.

16. Two ships sail from the same port; one sails due east 50 miles, and the other due south 84 miles; how far are they from each other? Ans. 97.75 miles.

17. A ladder or pole, 40 feet long, placed in the middle of a street, will reach a window of a house on each side of the street 24 feet from the pavement; what is the width of the street? Ans. 64 feet wide.

## THE CUBE ROOT.

The cube root of a given number, is such a number as being multiplied by itself, and then into that product, produces the given number.

### RULE.

1. Point off the sum into periods of three figures each, beginning with units.

2. Find the greatest cube in the left hand period, place the root of it in the quotient, subtract the cube from the left hand period, and to the remainder bring down the next period for a resolvend.

3. Square the quotient and multiply the square by 3 for a defective divisor.

4. Seek how often the defective divisor is contained in the resolvend, omitting the units and tens, or two right hand figures. Place the result in the quotient, and its square to the right of the divisor, supplying the place of tens with a cypher, whenever the square is less than ten.

5. Multiply the last figure of the quotient or root by all the figures in it previously ascertained; multiply that product by 30, and add their product to the divisor, to complete it.

6. Multiply and subtract as in Simple Division, and to the remainder bring down the next period, for a new resolvend. Find a divisor as before, and thus proceed until all the periods are brought down.

*Note.*—The cube root of a vulgar fraction is found by reducing it to its lowest terms, and extracting, as in the square root; and if the fraction be a surd, reduce it to a decimal, and then extract the root.

In extracting the cube root, if the sum be in part decimals, or if the whole be decimals, point the figures as in the square root, observing to have three figures in a period instead of two; and in all cases in the cube root, when there is a remainder, if it be required to obtain decimal figures to the root, proceed as directed in the square root, only add three cyphers, in place of two, to the remainder.

## PROOF.

Involve the root to the third power, adding the remainder, (if any,) to the result.

## EXAMPLES.

1. What is the cube root of 182284263?

Defec. div. {	$5 \times 5 \times 3 = 7536$	Root. 182284263(567
& sq. of 6. }	$6 \times 5 \times 30 = 900$	125
	Complete divisor—8436	—
	$56 \times 56 \times 3 = 940849$	)57284 resolv.
	$7 \times 56 \times 30 = 11760$	50616
	Complete divisor 952609	—
		)6668263
		6668263

2. What is the cube root of 48228.544? Ans. 36.4.

3. What is the cube root (or 3d root) of 2? Ans. 1.259921.

4. What is the cube root of 132651? Ans. 51.

5. What is the cube root of 4173281? Ans. 161.

6. What is the cube root of .008649? Ans. .2052+.

7. What is the cube root of  $\frac{125}{8}$ ? Ans.  $\frac{5}{2}$ .

8. What will be the cube root of 160, the decimal being continued to three places? Ans. 5.428+.

9. If the contents of a globe amount to 5832 cubick inches, what are the dimensions of the side of a cubick block containing the same quantity? Ans. 18 in. square.

10. If the diameter of the planet Jupiter is 12 times as much as the diameter of the earth, how many globes of the earth would it take to make one as large as Jupiter? Ans. 1728.

11. If the sun is 1000000 times as large as the earth, and the earth is 8000 miles in diameter, what is the diameter of the sun? Ans. 800000 miles.

*Note.*—The roots of most powers may be found by the square and cube roots only; thus the square root of the square root is the biquadrate, or 4th root, and the sixth root is the cube of this square root.

*Questions concerning the powers and roots.*

1. What is called a power?
2. What power is the square?   Ans. The 2d power.
3. What is the cube of a number called?
4. How do you raise the power of a vulgar fraction?
5. What is the root of a power?
6. What is meant by extracting the square root?
7. Repeat the rule for doing it.
8. How do you proceed when the sum consists in part, or altogether, of decimals?
9. How do you extract the root of a vulgar fraction?
10. How do you proceed when the fraction is a surd?
11. What do you understand by the cube root?
12. Repeat the rule for extracting it.
13. How are sums in the square root proved?
14. How are sums in the cube root proved?

—  
**ALLIGATION.**

Alligation is a rule for mixing simples of different qualities, in such a manner that the composition may be of a mean or middle quality.

**CASE I.**

*To find the mean price of any part of the mixture, when the quantities and prices of several things are given.*

**RULE.**

As the sum of the quantities is to any part of the composition, so is the price of the quantities to the price of any particular part.

**EXAMPLES.**

1. A trader mixes 60 gallons of wine at 100 cents per gallon; 40 gallons, at 80 cents; and 30 gallons of water. What should be the price per gallon?

	<i>gals.</i>	<i>cts.</i>	\$
Wine	60	at 100	=60.00
Wine	40	at 80	=32.00
Water	30		

<i>gals.</i>	<i>gal.</i>	\$	
130	:	1	: : 92.00.   Ans. 70 $\frac{1}{3}$ .

2. A trader mixes a quantity of tea as follows, viz:—  
4 lbs. of tea at 42 cents per lb.; 6 lbs. at 33 cents; 12  
lbs. at 75 cents, and 15 lbs. at 80 cents. What can he  
sell it for per lb.?

Ans.  $66\frac{3}{4}$  cents.

3. A farmer mixes 20 bushels of wheat at 5s. per  
bushel, with 35 bushels of rye at 3s, and 40 bushels of  
barley at 2s. per bushel; how much is a bushel of the  
mixture worth?

Ans. 3s.

### CASE II.

*When the prices of several simples are given to find what  
quantity of each, at their respective prices, must be taken  
to make a compound at a proposed price.*

### RULE.

Set the prices of the simples in a column under each  
other. Connect with a continued line, the rate of each  
simple which is less than that of the compound, with one  
or any number of those that are greater than the com-  
pound, and each greater rate, with one or more of the  
less. Place the difference between the mixture rate,  
and that of each of the simples, opposite to the rates,  
with which they are linked. Then, if only one differ-  
ence stand against any rate, it will be the quantity be-  
longing to that rate; but if there be several, their sum  
will be the quantity. Different modes of linking, will  
produce different answers.

### EXAMPLES.

1. A merchant would mix wines at 17s. 18s. and 22s.  
per gallon, so that the mixture may be worth 20s. per  
gallon: what quantity of each must be taken?

Mixture	$\left\{ \begin{array}{l} 17 \\ 18 \\ 22 \end{array} \right\}$	2 at 17s.
rate 20s.		2 at 18s.
		$3 + 2 = 5$ at 22s.

Ans. 2 gallons at 17s., 2 at 18s., and 5 at 22s.

2. How much barley at 40 cents, corn at 60, and  
wheat at 80 cents per bushel, must be mixed together,  
that the compound may be worth  $62\frac{1}{2}$  cents per bushel?

Ans.  $17\frac{1}{2}$  bush. of barley,  $17\frac{1}{2}$  of corn, and 25 of wheat.



CASE III.

*When the price of all the simples, the quantity of one of them, and the mean price of the mixture, are given, to find the quantities of the other simples.*

RULE.

Find an answer as before, by connecting; then, as the difference of the same denomination with the given quantity, is to the differences respectively, so is the given quantity, to the different quantities required.

EXAMPLES.

1. How much gold of 15, 17, 18, and 22 carats fine must be mixed together to form a composition of 40 oz. of 20 carats fine?

Mean or Mixture rate. 20	$\left\{ \begin{array}{l} 15 \\ 17 \\ 18 \\ 22 \end{array} \right\}$	-	-	-	2
		-	-	-	2
		-	-	-	2
		5+3+2=10			10

Then as 16 : 2 : : 40 : 5 } Answer. 16  
and as 16 : 10 : : 40 : 25 }

Ans. 5 oz. of 15, 17 and 18 carats fine, and 25 oz. of 22 carats fine.

2. A grocer has currents at 4d., 6d., 9d., and 11d., per lb. and he would make a mixture of 240 lbs. that might be sold at 8d. per lb.; how much of each kind must he take?

Ans. 72 lbs. at 4d., 24 at 6d., 48 at 9d. and 96 at 11d.

CASE IV.

*When the prices of the simples, the quantity to be mixed, and the mean price are given, to find the quantity of each simple.*

RULE.

Connect the several prices, and place their differences as before; then, as the sum of the differences thus given, is to the difference of each rate, so is the quantity to be compounded, to the quantity required.

## EXAMPLES.

How much sugar at 9 cents, 11 cents, and 14 cents per lb. will be necessary to form a mixture of 20 lbs. worth 12 cents per lb.?

$$12 \left\{ \begin{array}{l} 9 \\ 11 \\ 14 \end{array} \right\} \begin{array}{r} 2 \\ 2 \\ 3+1=4 \end{array} \quad \begin{array}{r} 2 \\ 2 \\ 8 \end{array}$$

Then, as  $8 : 2 :: 20 : 5$  lbs. 9 cents. }  
 $8 : 2 :: 20 : 5$  lbs. 11 cents. } Answer.  
 $8 : 4 :: 20 : 10$  lbs. 14 cents. }

2. A grocer has sugar at 24 cents per lb. and at 13 cents per lb.; and he wishes so to mix 2 cwt. of it, that he may sell it at 16 cents per lb.; how much of each kind must he take? Ans.  $162\frac{1}{2}$  lbs. of that at 13 cents, and  $61\frac{1}{2}$  lbs. of that at 24 cents.

3. How many gallons of water must be mixed with wine worth 60 cents per gallon, so as to fill a vessel of 80 gallons, that may be sold at  $41\frac{1}{2}$  cents per gallon?

Ans. 25 gallons of water, and 55 of wine.

## POSITION.

Position is a rule for solving questions, by one or more supposed numbers. It is divided into two parts, namely single and double.

## SINGLE POSITION.

Single position teaches to solve questions which require but one supposition.

## RULE.

Suppose a number, and proceed with it as if it were the real one, setting down the result—Then, as the result of that operation, is to the number given, so is the supposed number, to the number sought.

## EXAMPLES.

1. What number is that, which being multiplied by 7 and the product divided by 6, will give 14 for the quotient? Suppose 18

$$\begin{array}{r} 7 \\ 6 \overline{)126} \end{array}$$

Then, as  $21 : 14 :: 18 : 12$  Answer.

2. What number is that, of which one half exceeds one third by 15?

$$\text{Suppose } 60 \text{—Then } \frac{1}{2} | 60 | \frac{1}{3} | 60$$

$$\frac{30}{30} \quad \frac{20}{20}$$

$$\text{Subtract } 20$$

$$\frac{10}{10}$$

Then, as 10 : 15 :: 60 : 90 Answer.

3. What number is that, which being increased by  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{4}$  of itself, the sum will be 125? Ans. 60.

4. A schoolmaster being asked how many scholars he had, answered, that if  $\frac{2}{3}$  of his number were multiplied by 7, and  $\frac{2}{3}$  of the same number added to the product, the sum would be 292. What was his number?

Ans. 60.

5. A schoolmaster being asked what number of scholars he had, said, if I had as many, half as many, and one fourth as many more, I should have 99. What was number?

Ans. 36.

6. A person, after spending  $\frac{1}{2}$  and  $\frac{1}{3}$  of his money, had \$30 left; what had he at first?

Ans. \$180.

7. Seven eighths of a certain number exceed four fifths by 6. What is that number?

Ans. 80.

8. A certain sum of money is to be divided among 4 persons, in such a manner that the first shall have  $\frac{1}{3}$  of it, the second  $\frac{1}{4}$ , the third  $\frac{1}{5}$ , and the fourth the remainder, which is \$28; what is the sum?

Ans. \$112.

9. What sum, at 6 per cent. per annum, will amount to £860, in 12 years?

Ans. £500.

10. A person having about him a certain number of crowns, said, if a third, a fourth and a sixth of them were added together, the sum would be 45; how many crowns had he?

Ans. 60.

11. What is the age of a person who says, that if  $\frac{1}{4}$  of the years he has lived be multiplied by 7, and  $\frac{2}{3}$  of them be added to the product, the sum would be 292?

Ans. 60 years.

12. What number is that, which being multiplied by 7, and product divided by 6, the quotient will be 14?

Ans. 12.

**DOUBLE POSITION.**

Double Position teaches to resolve questions by means of two supposed numbers.

**RULE.**

Suppose two convenient numbers, and proceed with each according to the condition of the question, and set down the errors of the results. Multiply the errors into their supposed numbers, crosswise; that is, multiply the first supposed number by the last error, and the last supposed number by the first error.

If the errors be alike, that is, both too much, or both too little, divide the difference of their products by the difference of the errors—the quotient will be the answer. But if the errors be unlike, that is, one too large and the other too small, divide the sum of the products by the sum of the errors.

**EXAMPLES.**

1. What number is that, whose  $\frac{1}{3}$  part exceeds the  $\frac{1}{4}$  part by 16?

Suppose 24; and as  $\frac{1}{3}$  of 24 is 8, and  $\frac{1}{4}$  of it is 6, it is evident that the third part exceeds the fourth part by 2 instead of 16; and therefore the error is 14 too small. Again, suppose 48; and  $\frac{1}{3}$  of 48 being 16, and  $\frac{1}{4}$  being 12, it is manifest that the third part exceeds the fourth by 4, instead of 16; hence the error is 12 too small.—Then, the errors being alike, proceed thus—

1. supposition 24	er.	14 too small.
2. supposition 48	er.	12 too small.
14	672 product.	288 product.
12	288	

2 dif. of er. 2) 384 difference of the products.

192 Answer.

2. A son asking his father how old he was, received this answer: Your age is now  $\frac{1}{4}$  of mine; but 5 years ago, your age was  $\frac{1}{5}$  of mine. What are their ages?

Ans. 20 and 80.

3. Two persons, A and B, have each the same income. A saves  $\frac{1}{3}$  of his; but B, by spending 50 dollars per annum more than A, finds himself at the end of 4 years one hundred dollars in debt. What was their income, and what did each spend?

Ans. Their income was \$125 per annum for each. A spends \$100 and B spends \$150 per annum.

4. What number, added to the sixty-second part of 7626, will make the sum of 200? Ans. 77.

5. A man being asked how many sheep he had in his drove, said, if I had as many more, half as many more, one fourth as many more, and  $12\frac{1}{2}$ , I should have 40.—How many had he? Ans. 10.

6. An officer had a division,  $\frac{1}{2}$  of which consisted of English soldiers,  $\frac{1}{4}$  of Irish,  $\frac{1}{6}$  of Canadians, and 50 of Indians. How many were there in the whole?

Ans. 600.

7. A servant being hired for 30 days, agreed to receive 2s. 6d. for every day he laboured, and to forfeit 1s. for every day he played. At the end of the term his pay amounted to £2. 14s. How many of the days did he labour? Ans. 24.

8. What number is that, which being multiplied by 6, the product increased by adding 18 to it, and the sum divided by 9, the quotient will be 20? Ans. 27.

## ARITHMETICAL PROGRESSION.

Arithmetical Progression is a series of numbers increasing or decreasing by a common difference; as, 1, 2, 3, 4, 5; 1, 3, 5, 7, 9; 5, 4, 3, 2, 1; 9, 7, 5, 3, 1, &c. The numbers in a series are called terms—the first and last terms are called extremes, and the common difference is the number by which the terms in a series differ from each other, as in 2, 5, 8, 11, &c.—the common difference is 3.

In any series in Arithmetical Progression, the sum of the two extremes is equal to the sum of any two terms, equally distant from them, or equal to double the middle term when there is an uneven number of terms in

the series. Thus, in the series 2, 4, 6, 8, 10, 12,—the extremes are 2 and 12, equal to 14, and if you add 10 and 4, or 8 and 6, the result will be the same; and in the series 2, 4, 6, 8, 10, the extremes are 10 and 2, and as the number of terms is uneven 6 is the middle one, which, when doubled makes 12, and the extremes when added together make the same amount.

### CASE I.

*The first term, common difference, and number of terms, being given, to find the last term and sum of all the terms.*

### RULE.

Multiply the common difference by one less than the number of terms, and to the product add the first term, the sum will be the last. Add the first and last terms together, multiply their sum by the number of terms, and half the product will be the sum of all terms.

### EXAMPLES.

1. The first term in a certain series is 3, the common difference 2, and the number of terms 9; to find the last term, and the sum of all the terms.

One less than the number of terms is 8.

2 common difference.

8 number of terms less one.

---

16 product.

3+ first term.

---

19 last term.

3+ first term.

---

22

9× number of terms.

---

2)198

---

Answer 99 sum of all the terms.

2. A person sold 80 yards of cloth at 3 cents for the first yard, 6 for the second, and thus increasing 3 cents every yard; what was the whole amount? Ans. \$97.20.

3. How many times does a clock usually strike in 12 hours? Ans. 78.

4. A man on a journey travelled 20 miles the first day, 24 the second, and continued to increase the number of miles by every day for 10 days. How far did he travel? Ans. 380 miles.

5. A farmer bought 20 cows, and gave 2 dollars for the first, 4 for the second, and so on, giving in the same proportion from the first to the last. What did he give for the whole? Ans. \$120.

### CASE II.

*When the two extremes and the number of terms are given to find the common difference.*

#### RULE.

Subtract the less extreme from the greater, and divide the remainder by one less than the number of terms—the quotient will be the common difference.

#### EXAMPLES.

1. The extremes being 3 and 19, and the number of terms 13, what is the common difference?

9	19	11		
1	3	13	num. of terms	
—	—	1	80	}
8	8)16	—	20	
—	—	12	—	
Ans. 2		12) 80		div. of extremes

Common difference 5 Answer.

3. If the extremes be 10 and 70, and the number of terms 21, what is the common difference, and the sum of the series? Ans. com. diff. 3, and the sum, 840.

4. A certain debt can be paid in one year, or 52 weeks, by weekly payments in Arithmetical Progression, the first payment being 1 dollar, and the last 103 dollars. What is the common difference of the terms? Ans. \$2.

5. A debt is to be discharged at 16 several payments in Arithmetical Progression; the first payment to be 20 dollars, and the last 110 dollars. What is the common difference? Ans. \$5.

## GEOMETRICAL PROGRESSION.

Geometrical Progression is the increase of any series of numbers by a common multiplier, or the decrease of any series by a common divisor; as 3, 6, 12, 24, 48; and 48, 24, 12, 6, 3. The multiplier or divisor by which any series is increased or decreased, is called the ratio.

### CASE I.

*To find the last term and sum of the series.*

#### RULE

Raise the ratio to a power whose index is one less than the number of terms given in the sum. Multiply the product by the first term, and the product of that multiplication will be the last term: then multiply the last term by the ratio, subtract the first term from the product, and divide the remainder by a number that is one less than the ratio—the quotient will be the sum of the series.

#### EXAMPLES.

1. Bought 12 yards of calico, at 2 cents for the first yard, 4 cents for the second, 8 for the third, &c.: what was the whole cost?

NOTE.—The number of terms 12, and the ratio 2.

1st.	1024	10th. power.	
term 2 1st. power.	2		
2	—		
4 2d. power.	2048	11th power, or one less	
2	2	2st term	} than the number of terms.
8 3d. power.	—		
2	4096		
16 4th. power.	2	the ratio.	
2	—		
32 5th. power.	8192		
32	—	2 subtract the 1st. term.	
64	1) 8190	1 is one less than the	
96	—	[ratio.	
1024 10th. power.	\$81.90	Answer.	

2. Bought 10 lbs. of tea, and paid 2 cents for the first pound, 6 for the second, 18 for the third, &c. What did the whole cost?  
Ans. \$590.48.



3. The first term in a sum is 1, the whole number of terms 10, and the ratio 2; what is the greatest term, and the sum of all the terms? Ans. The greatest term is 512, and the sum of the terms 1023.

4. What debt may be discharged in 12 months, by paying 1 dollar the first month, 2 dollars the second month, 4 the third month, and so on, each succeeding payment being double the last; and what will be the amount of the last payment?

Ans. The debt is \$1095, and the last payment \$2048.

5. A father whose daughter was married on a new-year's day, gave her one cent, promising to triple it on the first day of each month in the year; what was the amount of her portion?

Ans. \$2657.20.

6. One Sessa, an Indian, having invented the game of chess, shewed it to his prince, who was so delighted with it, that he promised him any reward he should ask; upon which Sessa requested that he might be allowed one grain of wheat for the first square on the chess board, 2 for the second, 4 for the third, and so on, doubling continually, to 64, the whole number of squares.—Now, supposing a pint to contain 7680 of these grains, and one quarter or 8 bushels to be worth 27s. 6d., it is required to compute the value of all the wheat?

Ans. £34481488206.

7. What sum would purchase a horse with 4 shoes, and eight nails in each shoe, at one farthing for the first nail, a half penny for the second, a penny for the third, &c., doubling to the last?

Ans. £1473924. 5s. 3½d.

8. A merchant sold 15 yards of satin, the first yard for 1s. the second for 2s. the third for 4s. the fourth for 8s. &c.; what was the price of the 15 yards?

Ans. £1638. 7s.

9. Bought 30 bushels of wheat, at 2d. for the first bushel, 4d. for the second, 8d. for the third, &c.; what does the whole amount to, and what is the price per bushel on an average?

Ans. { £2947848. 10s. 6d. Amount.  
£298231. 12s. 4d. per bushel.

## PERMUTATION.

Permutation is used to show how many ways things may be varied in place or succession.

### RULE.

Multiply all the terms of the series continually, from 1 to the given number inclusive; and the last product will be the answer required.

### EXAMPLES.

1. How many changes can be made with 8 letters of the alphabet?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320 \text{ Answer.}$$

2. In how many different positions can 12 persons place themselves round a table?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479001600 \text{ Ans.}$$

3. How many changes may be made with the alphabet?  
Ans. 620448401733239439360000.

## SKETCH OF MENSURATION, OR PLANES AND SOLIDS.\*

Planes, surfaces, or superficies, are measured by the inch, foot, yard, &c., according to the measures used by different artists. A superficial foot is a plane or surface of one foot in length or breadth, without reference to thickness. Solids are measured by the solid inch, foot, yard, &c.; thus, 1728 solid inches, that is  $12 \times 12 \times 12$  make one cubick or solid foot. Solids include all bodies which have length, breadth and thickness.

### ARTICLE I.

*To measure a square having equal sides.*

### RULE.

Multiply any one side of the square by itself, and the product will be the area, or superficial contents, in feet, yards, or any other measure, according to the measure used in measuring the sides.

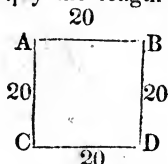
\*Planes are the same as superficies, or surfaces.

EXAMPLES.

Let A, B, C and D represent a square, having equal sides each measuring 20 feet. Multiply the length of one side by itself, thus—

20 feet  
20 feet

Ans. 400 square feet.



2. How many square rods are there in a field 90 rods square? Ans. 8100 square rods.

ARTICLE II.

*To measure the plane or surface of a parallelogram.*

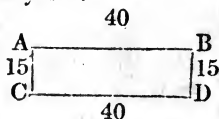
RULE.

Multiply the length by the breadth—the product will be the superficial contents.

EXAMPLE.

Let A, B, C and D represent a parallelogram whose length is 40 yards, and breadth 15 yards.

Breadth 15 yards  
Length 40 yards



Ans. 600 square yards.

2. How many square feet are there in the floor of a room 36 feet long and 16 feet wide? Ans. 576 square ft.

3. I engage to give a plasterer 15 cents per square yard, for plastering the walls and ceiling of a room 30 feet long, 15 feet wide, and 9 feet high. How much will his work come to? Ans. \$21.00.

*Note.*—The contents of boards and other articles which are measured by feet, &c., may be easily found by Duodecimal Fractions.

ARTICLE III.

*To measure the plane or surface of a triangle.*

RULE.

Multiply the base by half the perpendicular, if it be a right angled triangle, and the product will be the area, or superficial contents; or multiply the base and perpendicular together, and half the product will be the area.

But if it be an oblique angled triangle, multiply half the length of the base by a perpendicular let fall on the base from the angle opposite to it, and the product will be the area.

## EXAMPLES.

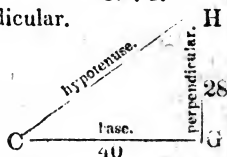
1. Let C, H and G represent a right angled triangle, having the right angle at G; the base C G being 40 feet, and the perpendicular H G, 28 feet.

No. 1.

14 feet, or half the perpendicular.

40 feet, or the base.

560 feet—the area.



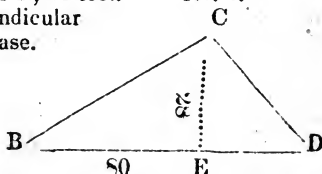
2. Let B, C and D represent an oblique angled triangle; the length of the base B D being 80 feet, and the perpendicular C E, 28 feet.

No. 2.

28 the perpendicular

40 half the base.

1120 Answer.



3. How many square rods are there in a triangular field, one of the corners of which is a right angle, and one of the shorter sides of which is 36 rods, and the other 24 rods?

Ans 432 square rods.

NOTE.—Right angled triangles are such as have one angle like the corner of a square, and which is called the right angle, containing 90 degrees; as the angle G in the triangle, No. 1.—Oblique angled triangles are such as have each of the angles, either more or less than 90 degrees, as in the triangle, No. 2.

## ARTICLE IV.

To measure a circle.

Note.—Circles are round figures, bounded every where by a circular line, called the periphery, and also the circumference. A line passing through the centre

is called the diameter. Half the length of the diameter is called the radius.

The diameter may be found by the circumference, thus—As 22 is to 7 so is the circumference to the diameter; and in like manner may the circumference be found by the diameter; for, as 7 is to 22, so is the diameter to the circumference.

1. What is the diameter of a wheel, or circle, whose circumference is 16 feet?      Ans. 5 feet, nearly.

2. What is the circumference of a circle, whose diameter is 20 feet?      Ans. 63 feet nearly.

3. If the distance through the earth be 8000 miles, how many miles around it?      Ans. 25143 miles nearly.

#### ARTICLE V.

*To find the superficial contents, or area, of a circle.*

##### RULE.

Multiply half the circumference by half the diameter, and the product will be the answer. Or, multiply the square of the diameter by .7854; or multiply the square of the circumference by .07958, and in either case the product will be the answer.

##### EXAMPLE.

How many square feet are contained in a circle whose circumference is 44 feet, and whose diameter is 14 feet?

22 half the circumference.

7 half the diameter.

---

154 square feet.      Answer.

The same may be done by multiplying the diameter and circumference together, and dividing the product by 4, thus,  $44 \times 14 = 616 \div 4 = 154$ .      Answer.

2. How many square feet are there in the area of a circle whose circumference is 16 feet?

Ans. 20 square feet.

3. How many square feet are there in the area of a circle whose diameter is 20 feet?      Ans. 315 square ft.

#### ARTICLE VI.

*To measure the surface of a globe or sphere.*

##### RULE.

Multiply the circumference by the diameter, the product will be the surface, or arc.

## EXAMPLES.

1. What are the superficial contents of a globe whose circumference is 220 feet, and whose diameter is 70 feet?  $220 \times 70 = 15400$  square feet. Answer.

2. How many square miles are contained on the surface of the whole earth, or globe, which we inhabit?

The circumference of the earth is estimated to be 25020 miles, and the diameter, 7964, nearly.

Then,  $25020 \times 7964 = 199259280$  square miles. Ans.

## ARTICLE VII.

*To find the solid contents of a cube.\**

## RULE.

Multiply the length of one side by itself, and multiply the product by the same length, that is, by the same multiplier; the last product will be the solid contents of the cube.

## EXAMPLES.

1. How many solid feet are contained in a cube, or solid block of 6 equal sides, each side being 3 feet in length, and 3 in breadth?

$3 \times 3 \times 3 = 27$  solid or cubic feet. Ans.

When the contents are required of right angled solids, whose length, breadth, &c., are not equal; multiply the length by the breadth, and that product by the thickness, the product will be the answer.

2. Required the contents of a load of wood, whose length is 8 feet, breadth 4 feet, and height or thickness 1 foot.  $8 \times 4 \times 1 = 32$  solid feet, or 1 cord. Ans.

3. Required the contents of a stone  $16\frac{1}{2}$  feet in length,  $1\frac{1}{2}$  in breadth, and 1 foot in thickness.

$16.5 \times 1.5 \times 1 = 24.75$  solid feet, or 1 perch. Ans.

*Note.*—Solids whose dimensions are in feet or inches, are more easily measured by Duodecimals.

## ARTICLE VIII.

*To find the contents of a prism.*

A prism is an angular figure, generally of three equal sides, whose ends are in the form of triangles. It resembles a pile of three sides, whose whole length is of equal ligness.

\*A cube is a solid body of equal sides, each of which is an exact square.

## RULE.

Find the area or superficial contents of one end as of any other triangle, then multiply the area by the length of the prism, and the product will be the solidity.

## EXAMPLE.

What are the solid contents of a prism, the sides of the triangles of which measure 13 inches, the perpendicular extending from one of its angles to its opposite side, 12 inches, and its length 18 inches?

$$13 \times 12 = 156 \div 2 = 78 \times 18 = 1404 \text{ cubick inches. Ans.}$$

## ARTICLE IX.

*To find the contents of a cylinder.*

A cylinder is a long round body, all its length being of equal bigness, like a round ruler.

## RULE.

Find the area of one end, by the rule for finding the area of a circle, then multiply it by the length, and the product will be the answer.

## EXAMPLE.

What is the solidity of a cylinder, the area of one end of which contains 2.40 square feet and its length being 12.5 feet?

$$2.40 \times 12.5 = 30 \text{ solid feet. Ans.}$$

## ARTICLE X.

*To find the solid contents of a round stick of timber, which is of a true taper from the larger to the smaller end.*

## RULE.

Find the area of both ends; add the two areas together and reserve the sum; multiply the area of the larger end by the area of the smaller end, extract the square root of the product, add the root to the reserved sum, then multiply this sum by one third the length of the stick, and the product will be the solidity.

NOTE.—As this method requires considerable labour, the following has been preferred for common use, though not quite so accurate.

## RULE.

Girt the stick near the middle, but a little nearer to the larger than to the smaller end; this will give the circumference at that place. Find the diameter by the circumference; multiply the circumference and the diameter together; then multiply one fourth of the product by the length and the answer will be nearly the solid contents.

## EXAMPLES.

What is the solidity of a round stick of timber that is 10 feet long, and its circumference near the middle is 2.61 feet?

$$\begin{array}{l} \text{As } 22 : 7 : : 2.61 : .83 \text{ diameter.} \\ \text{cir. diam.} \qquad \qquad \qquad \text{length. feet.} \\ 2.61 \times .83 = 2.1663 \div 4 = .5415 \times 10 = 5.4150. \\ \text{Ans. } 5.4150 \text{ solid feet.} \end{array}$$

## ARTICLE XI.

*To find the solid contents of a globe.*

## RULE.

Multiply the cube of the diameter by .5236, the product will be the solid contents. Or, multiply the superficial contents, or surface, by one sixth part of the surface. Or, multiply the cube of the diameter by 11, and divide the product by 21—in either case the product will be the solidity.

## EXAMPLES.

1. What are the solid contents of a globe whose diameter is 14 inches?

$$14 \times 14 \times 14 = 2744 \times .5236 = 1436.7584$$

cubick inches. Ans

2. How many solid miles are contained in the earth, or globe, which we inhabit?

Suppose the diameter to be 7954 miles; then,  $7954 \times 7954 \times 7954 = 503218686664$  the cube of the earth's axis, or diameter; then,

$$503218686664 \times .5236 = 263485304337$$

cubick miles. Ans.

*Note.*—The solidity of a globe may be found by the circumference, thus—Multiply the cube of the circumference by .016887—the product will be the contents.

## PRACTICAL QUESTIONS.

1. A cannon ball goes about 1500 feet in a second of time. Moving at that rate, what time would it take in going from the earth to the sun; admitting the distance to be 100 millions of miles, and the year to contain 365 days, 6 hours?

Ans.  $10 \frac{4808}{13143}$  years.



2. A young man spent  $\frac{1}{4}$  of his fortune in 8 months,  $\frac{3}{4}$  of the remainder in 12 months more, after which he had £410. left. What was the amount of his fortune?

Ans. £956. 13s. 4d.

3. What number is that, from which if you take  $\frac{3}{4}$  of  $\frac{3}{4}$ , and to the remainder add  $\frac{7}{16}$  of  $\frac{1}{16}$ , the sum will be 10?

Ans.  $10\frac{121}{3246}$ .

4. What part of 3, is a third part of 2? Ans.  $\frac{2}{9}$ .

5. If 20 men can perform a piece of work in 12 days, how many will accomplish another thrice as large, in one fifth of the time?

Ans. 300.

6. A person making his will, gave to one child  $\frac{13}{10}$  of his estate, and the rest to another. When these legacies were paid, the one proved to be £600 more than the other. What was the worth of the whole estate?

Ans. £2000.

7. The clocks of Italy go on to 24 hours; how many strokes do they strike in one complete revolution of the index?

Ans. 300.

8. What quantity of water must be added to a pipe of wine, valued at £33, to bring the first cost to 4s. 6d. per gallon?

Ans.  $20\frac{2}{3}$  gallons.

9. A younger brother received £6300, which was  $\frac{7}{8}$  of his elder brother's portion. What was the whole estate?

Ans. £14400.

10. What number is that which being divided by 2, or 3, 4, 5, or 6, will leave 1 remainder, but which if divided by 7 will leave no remainder?

Ans. 721.

11. What is the least number that can be divided by the nine digits without a remainder?

Ans. 2520.

12. How many bushels of wheat, at \$1.12 per bushel, can I have for \$81.76?

Ans. 73.

13. What will 27 cwt. of iron come to, at \$4.56 per cwt.?

Ans. \$123.12.

14. When a man's yearly income is 949 dollars, how much is it per day?

Ans. \$2.60.

15. My factor sends me word he has bought goods to the value of £500. 13s. 6d. upon my account; what will his commission come to at  $3\frac{1}{2}$  per cent.?

Ans. £17. 10s. 5 $\frac{1}{2}$ d.

16. How many yards of cloth, at 17s. 6d per yard, can I have for 13 cwt. 2 qrs. of wool, at 14d. per lb.?

Ans. 100 yards,  $3\frac{1}{2}$  qrs.

17. There is a cellar dug that is 12 feet every way, in length, breadth, and depth; how many solid feet of earth were taken out of it?

Ans. 1728.

18. If  $\frac{2}{3}$  of an ounce cost  $\frac{7}{8}$  of a shilling, what will  $\frac{1}{4}$  of a lb. cost?

Ans. 17s. 6d.

19. If  $\frac{5}{8}$  of a gallon cost  $\frac{1}{2}$  of a £. what will  $\frac{3}{4}$  of a tun cost?

Ans. £105.

20. If  $\frac{3}{8}$  of a ship be worth £3740, what is the worth of the whole?

Ans. £3973. 6s. 8d.

21. What is the commission on \$2176.50, at  $2\frac{1}{2}$  per cent?

Ans. \$54.41 $\frac{1}{2}$ .

22. In a certain orchard  $\frac{1}{2}$  of the trees bear apples,  $\frac{1}{3}$  pears,  $\frac{1}{4}$  plums,  $\frac{1}{5}$  of the apples, and 40 cherries; how many trees are in the orchard?

Ans. 1200.

23. If A travel by mail at the rate of 8 miles an hour, and when he is 50 miles on his way, B start from the same place that A did, and travel on horseback the same road at 10 miles an hour, how long and how far will B travel to come up with A?

Ans. 25 hours, and 250 miles.

24. Bought a quantity of cloth for 750 dollars,  $\frac{3}{8}$  of which I found to be inferior which I had to sell at 1 dollar 25 cents per yard, and by this I lost 100 dollars: what must I sell the rest at per yard that I shall lose nothing by the whole?

Ans. \$3.15 $\frac{1}{2}$ .

25. If the earth goes round the sun once in 365 days, 5 hours, 48 minutes, 49 seconds, and its distance from the sun 95000000 miles, what must be the distance of the planet Mercury from the Sun, admitting the time of its revolution round the Sun to be 87 days, 23 hours, 15 minutes, 40 seconds?

*Note.*—The planets describe equal areas in equal times therefore, as the square of the time of the revolution of one planet, round the Sun, is to the square of the time of the revolution of any other planet, so is the cube of the distance of one planet from the Sun, to the cube of the distance of any other from the Sun.

A SHORT SYSTEM  
OF  
**BOOK-KEEPING.**  
FOR  
FARMERS AND MECHANICS.

---

**BOOK-KEEPING** is the method of recording business transactions. It is of two kinds—single and double entry; but we shall only notice the former.

Single entry is the simplest form of Book-Keeping, and is employed by retailers, mechanics, farmers, &c. It requires a Day-Book, Leger, and where money is frequently received and paid out, a Cash-Book.

A few examples only are here given, barely sufficient to give the learner a view of the manner of keeping books; it being intended that the pupil should be required to compose similar ones, and insert them in a book adapted to this purpose.

The *Day-Book* contains entries of the several articles in the successive order of their dates. Each person must be made *Dr.* for what he receives, and *Cr.* by what is received of him on account.

Every month, or oftener, the *Day-Book* should be copied or posted into the *Leger*, as hereafter directed. The crosses, on the left hand column, show that the charge or credit, against which they stand, is posted, and the figures show the page of the *Leger* where the account is posted. Some use the figures only as post marks.

The *Leger* is the grand book of accounts, in which every person's account is collected from different parts of the *Day Book*, and inserted in one place; the *Dr.* and *Cr.* fronting each other on opposite pages or on opposite sides of the same page, which shows the whole state of the account at once.

Post or transfer the entries from the *Day Book* to the *Leger*, thus: Open an account in the *Leger* for the first person who stands *Dr.* or *Cr.* in the *Day Book*, *i. e.* write his name with *D.* on the left hand page of the folio, and *Cr.* on the right.

January 18th, 1831.

1 X	<b>Peter Simpson of Cin.</b>	<b>Dr.</b>	\$ C.
	To 15 yards of fine Broad-cloth, a 5.00	\$75.00	
	" 24 do. superfine do. a 7.75 . . .	186.00	
			261 00
1 X	<b>R. Fulton, of Newport,</b>	<b>Dr.</b>	
	To 1 gall. Molasses . . . . .	\$ 50	
	" 6 lbs. Coffee, a 37½ cts. . . . .	2.25	
	" 20 lbs. Sugar, a 10 cts. . . . .	2.00	
			4 75
	19		
1 X	<b>John Cathell, Carpenter,</b>	<b>Dr.</b>	
	To 16 yards Calico, a 12½ cts. . . . .	\$2.00	
	" 10 " Muslin, a 15 cts. . . . .	2.50	
	" 1 Vest pattern, . . . . .	.75	
	" 1 pair Gloves, . . . . .	.62½	
	" 25 lbs. Nails, a 8 cts. . . . .	2.00	
			7 87½
1 X	<b>Charles H. Grover,</b>	<b>Dr.</b>	
	To 25 Reams post paper, a \$3.00, . . .	\$75.00	
	" 18 " foolscap do. a 3.25, . . .	58.50	
			133 50
1 X	<b>George Whipple, Jr.</b>	<b>Dr.</b>	
	To 300 lbs. Pork, a 5 cts. . . . .	\$15.00	
	" 50 bu. Corn, a 20 cts. . . . .	10.00	
			25 00
	20		
1 X	<b>William Jones,</b>	<b>Dr.</b>	
	To 35 lbs. Iron, a 7 cts. . . . .	\$2.45	
	" Cash paid his order to John Baker,	1.25	
			3 70
1 X	<b>James L. Rowan,</b>	<b>Dr.</b>	
	To 50 bls. Flour, a \$3.50, . . . . .	\$175.00	
	" 25 bu. Potatoes, a 30 cts. . . . .	7.50	
	" 4 bls. Cider, a 1.50, . . . . .	6.00	
	" 75 lbs. Beef, a 4 cts. . . . .	3.00	
			191 50
1 X	<b>Peter Simpson,</b>	<b>Cr.</b>	
	By 30 cords Wood, a \$2.25, . . . . .	\$67.50	
	" 90 bu. Oats, a 12½ cts. . . . .	11.25	
	" 5 tons Hay, a 15.00, . . . . .	75.00	
			153 75

January 21st. 1831.

1 X	<b>Burchel J. Barney</b>	Dr.	\$ C.
	To 6 galls. Port wine, a \$2.50, . . .	\$15.00	
	" 216 lbs. Sugar, a 9 cts. . . . .	19.33	
	" 1 lb. Tea, . . . . .	1.25	
		—	35 69

1 X	<b>Claudius C. Williams,</b>	Dr.	
	To 172 bls. Whiskey 5190 galls. a 20 cts.	\$1038.00	
	" Paid his order in favor of Thomas and White, for salt, . . . . .	234.00	
		—	1272 00
		Cr.	
	By 750 lbs. Feathers, a 25 cts. . . . .	\$187.50	
	" Cash, . . . . .	500.0	
		—	687 50

—22—

1 X	<b>John Phares,</b>	Cr.	
	By sundries for which I gave my note at 60 days,		184 50

2 X	<b>Peter Simpson,</b>	Dr.	
	To order on Robert Fulton, . . . . .		1 37

2 X	<b>Robert Fulton,</b>	Cr.	
	By Peter Simpson's order on him, . . . .		1 37

2 X	<b>William Jones,</b>	Dr.	
	To 14 lbs. Veal, a 4 cts. . . . .	\$ .50	
	" 200 lbs. Flour, . . . . .	2.5	
		—	3 06
		Cr.	
	By his bill of blacksmith work, . . . . .		5 57

—23—

2 X	<b>John Cathell,</b>	Cr.	
	By 4 days carpenter work, a \$1.25, . . .	\$5.00	
	" 200 feet Poplar boards, a 37½ cts. . .	1.75	
		—	6 75

2 X	<b>George Whipple, Jr.</b>	Cr.	
	By Cash on account, . . . . .		20 00

2 X	<b>Claudius C. Williams,</b>	Cr.	
	By 10 bls Mackerel, No. 1, a \$12.00, . .	\$120.00	
	" 120 galls. Cog. Brandy, a 1.50. . .	180.00	
		—	300 00

2 X	<b>John Phares,</b>	Dr.	
	To cash on account of my note at 60 days,		75 00

January 24th, 1831.

1×	<b>Robert Fulton,</b>	Dr.	\$	C.
	To 1 piece Broadcloth containing 25 yards, a \$6 per yd.; 90 days' credit, . . . . .		150	00
1×	<b>Peter Simpson,</b>	Cr.		
	By Cash in full, . . . . .		108	62
2×	<b>Wm. Jones, Blacksmith,</b>	Dr.		
	To 217 lbs. Iron, a 8 cts. . . . .		17	36
		Cr.		
	By shoeing my horse, . . . . .		1	25
	25			
1×	<b>Charles H. Grover,</b>	Dr.		
	To rent of my house 3 months, a \$6 per month, Cr.		18	00
	By his bill for printing, . . . . .	\$12.62		
	" Cash, . . . . .	100.00		
			112	62
1×	<b>George Whipple, Jr.</b>	Dr.		
	To 1 Keg nails, weighing 215 lbs. a 8 cts. . . . .		17	20
	26			
2×	<b>James L. Rowan,</b>	Cr.		
	By use of his horse 20 days, a 50 cts. . . . .	\$10.00		
	" 1500 lbs. Rags, a 4 cts. . . . .	60.00		
	" 160 bu. Salt, a 50 cts. . . . .	80.00		
			150	00
2×	<b>Burchel J. Barney,</b>	Dr.		
	To 1 Bag of coffee weighing 216 lbs. a 16 cts. . . . .		34	56
3×	<b>C. Smith, of Lexington,</b>	Dr.		
	To 15 lbs. Wool, a 25 cts. . . . .	\$3.75		
	" 20 lbs. Flax, a 9 cts. . . . .	1.80		
			5	55
2×	<b>A. Dunn, of Columbus,</b>	Dr.		
	To 800 ft. Pine boards, a \$1.25, . . . . .	\$10.00		
	" 84 ft. Scantling, a 3 cts. . . . .	2.52		
			12	52
1×	<b>Robert Fulton,</b>	Cr.		
	By 1200 lbs. Pork, a 4 cts. . . . .		48	00
1×	<b>William Jones,</b>	Cr.		
	By Cash, . . . . .		17	30

January 27th. 1831.

			\$	C.
	<b>John Cathell,</b>	<b>Dr.</b>		
1×	To 350 lbs. Nails, a 8 cts. . . . .	\$28.00		
	" 17 Door locks, a 1.25, . . . . .	21.25		
			49	25
	<b>Charles H. Grover,</b>	<b>Cr.</b>		
1×	By Cash on account, . . . . .		10	37
	<b>Claudius C. Williams,</b>	<b>Dr.</b>		
2×	To 856 lbs. Tobacco, a 5 cts. . . . .		42	80
	<b>George Whipple, Jr.</b>	<b>Cr.</b>		
1×	By 1700 lbs. Cheese, a 7 cts. . . . .		119	00
	<b>James L. Rowan,</b>	<b>Dr.</b>		
2×	To 24 yds. Linen shirting, a 87½ cts. . . . .		21	00
	<b>John Cathell,</b>	<b>Cr.</b>		
1×	By Cash in full, . . . . .		50	37½
	28			
	<b>Calvin Smith,</b>	<b>Cr.</b>		
3×	By 41 lbs. Coffee, a 18 cts. . . . .	\$7.38		
	" 36 lbs. Sugar, a 12½ cts. . . . .	4.50		
			11	88
	<b>George Whipple, Jr.</b>	<b>Cr.</b>		
1×	By 12 days labour of self, a 62½ cts. . . . .	\$7.50		
	" 7 days do. of his horse, a 25 cts. . . . .	1.75		
			9	25
	<b>Burchel J. Barney</b>	<b>Cr.</b>		
2×	By cash in full, . . . . .		70	25
	<b>Andrew Dunn,</b>	<b>Dr.</b>		
2×	To mending Wagon, . . . . .	\$5.75		
	" Timber and materials for do. . . . .	1.25		
			7	00
	<b>James L. Rowan,</b>	<b>Cr.</b>		
2×	By Cash in full, . . . . .		62	50
	<b>John Phares,</b>	<b>Dr.</b>		
2×	To 730 lbs. Sea Island salt, a 15 cts. in full of my note. . . . .		109	50
	<b>Calvin Smith,</b>	<b>Dr.</b>		
3×	To 75 yds. Domestic cloth, a 50 cts. . . . .		37	50

## FORM OF A LEDGER.

[1]

Dr.				<i>Peter Simpson,</i>		Cr.			
1831.			\$	c.	1831.		\$	c.	
Jan. 18	1	To Sundries,	26	0	Jan. 20	1	By sundries,	153	75
" 22	2	" Order on R. F.	1	37	" 24	3	" Cash,	108	62
			26	37				262	37

Dr.				<i>Robert Fulton,</i>		Cr			
1831.			\$ c.	1831.			\$ c.		
Jan. 18	1	To Sundries,	4	75	Jan. 22	2	By Order P. S.	1	37
" 24	3	" Broadcloth	150	00	" 26	3	" Pork,	48	00
			154	75			" Balance,	105	8
								154	75
		" Balance, .	105	38					

Dr.				<i>John Cathell,</i>				Cr.			
1831.				\$	c.	1831.				\$	c.
Jan. 19	1	To Sundries,		7	87½	Jan. 22	2	By Sundries,		6	75
" 27	4	" do.		1	25	" 27	4	" Cash,		50	37½
				57	12½					57	12½

Dr.				<i>Charles H. Grover,</i>				Cr.			
1831.				\$	c.	1831.				\$	c.
Jan. 19	1	To Paper,		133	50	Jan. 25	3	By Sundries,		112	62
" 25	3	" Rent,		18	00	" 27	4	" Cash,		10	37
				—	—			" Balance,		28	51
				151	50					—	—
				—	—					151	50
		" Balance,		28	51					—	—

Dr.				<i>George Whipple, Jr.</i>				Cr.			
1831.			\$	c.	1831.			\$	c.		
Jan. 19	1	To Sundries	25	00	Jan. 23	2	By Cash,	20	00		
" 25	3	" Naives,	17	20	" 27	4	" Cheese,	119	00		
		" Balance	106	05	" 28	4	" Sundries,	9	25		
			---	---				---	---		
			148	25				148	25		
			---	---				---	---		
							" Balance,	106	05		





Dr.				Calvin Smith,				Cr.			
1831.				\$	c.	1831.		\$	c.		
Jan. 26	3	To Sundries,		5	55	Jan. 28	4	By Sundries,	11	88	
" 28	4	" Cloth,		37	50			" Balance,	31	15	
				—	—				—	—	
				43	05				43	05	
				—	—				—	—	
		" Balance,		31	15						

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## CASH-BOOK.

This book records the payments and receipts of cash.

It is kept by making cash *Dr.* to cash on hand and what is received, and *Cr.* by whatever is paid out.

At the end of every day or week, as may best suit the nature of the business, the cash on hand is counted, and entered on the *Cr.* side.

If there is no error, this will make the sum of the *Dr.* equal to that of the *Cr.* A balance is then struck, and the cash on hand carried again upon the *Dr.* side.

# FORM OF A CASH-BOOK.

179

Dr.

CASH.

Cr.

1831.		\$	c.	1831.		\$	c.
Jan. 1	To cash on hand	42	59	Jan. 1	By am't. paid		
2	" J. Ball	2	87		repairs		
"	" E. Jennison	10	00	3	" Paid note to		
3	" D. Roe paid				A. Weston	7	00
	acc't	35	5	5	" Paid workmen	24	00
"	" H. Austin on			6	" Family ex-		
	his note	3	10		penses		13
4	" S. Ball on acc't	3	75	"	" Merchandise		
5	" A. Higby	18	00		bought at		
6	" Sales of Mer-				auCTION	5	50
	chandise	31	31	"	Cash on hand	38	08
		85	58			35	08
	8 Cash on hand	36	8				

*Form of a Bill of parcels from the preceding Work.*  
**Mr. John Cathell**

To Solomon Thrifty, Dr.

1831.		\$	c.
Jan. 16.	To 16 yds. Calico, a 12½ cts. . . . .	\$2.00	
" "	" 10 yds. Muslin, a 15 cts. . . . .	1.50	
" "	" 1 Vest pattern, . . . . .	75	
" "	" 1 pair Gloves, . . . . .	62½	
" "	" 25 lbs. Nails a 8 cts. . . . .	2.00	
		6	87½
" 27	" 350 lbs. Nails a 8 cts. . . . .	28.00	
" "	" 17 Door locks a \$1.25 . . . . .	21.25	
		49	25
		56	12½
	Cr.		
" 5	By 4 days Carpenter work a \$1.25 . . .	\$5.00	
" "	" 200 ft. Poplar boards a 87½ cts. . . .	1.75	
" 27	" Cash in full, . . . . .	43.37½	
	Errors excepted.	56	12½

CINCINNATI, Jan. 27th, 1831.

SOLOMON THRIFTY.

*2d Form.*

**Claudius C. Williams,**

To Solomon Thrifty, Dr.

1831.		\$	c.
Jan. 21.	To 172 bls. Whiskey, 5190 galls. a 20		
	cts. . . . .	\$1038.00	
" "	" Paid his order favour of Thomas		
	& White, for Salt . . . . .	234.00	
		1272	00
" 27	" 856 lbs. Tobacco, a 5 cts. . . . .	42	80
		1314	80

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1831

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